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Qualitative analysis of impulsive state feedback control to an algae-fish system with bistable property

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ABSTRACT

A kind of consumer-resource system is proposed to describe the bidirectional interactions of the algae and the fish in an eutrophic water body. The dynamical properties of the proposed continuous system are given. For the bistable case, an impulsive semidynamical system with state feedback control, which depends on the biomass of the algae, is formulated and investigated to consider the feasibility of state feedback control for the aim of maintaining two species coexisting. The impulsive semidynamical system has three cases corresponding to three kinds of control measures: releasing fish, spraying algaecide, integrated control combining releasing fish and spraying algaecide. The existences of order-1 periodic solutions of three models are discussed by using successor function, respectively. The conditions under which the order-1 periodic solution is stable are given by using the Poincaré map and the analogue of Poincaré criterion. Mathematical results show that, for every one of three control strategies, there exists a range of control parameter in which the corresponding control is feasible. Finally, those mathematical results are verified by numerical simulations and the practical meanings are given.

1. Introduction

The algae can fix inorganic carbon through photosynthesis and make them into carbohydrates, which provides the basis of water productivity. As a producer in a fresh water ecological system, the algae is one of the food resource of the fish. But in an eutrophic water body, some kinds of algaes can grow quickly and release toxin into the water body, which have negative effects on the growth of the fish and other aquatic organisms. Therefore, the interactions of the fish and the algae are bidirectional. On the other hand, for the aims of protecting the fish and other aquatic organisms, it is necessary to control the biomass of the algae.

There are various of measures to decrease the biomass of the algae. Chemical measure is quick to remove the algae, but it can produce secondary pollution. It is generally believed that biological measure is safe. During the material transformation of food chain, every 1 kg fish needs to consumer about 100 kilograms of planktonic algae. Therefore, releasing some algophagous fish (e.g., Black carp, Grass carp) is one of the effective measures to control the excessive growth of algae in a fresh water. But if the fish is excessively released, plankton community will be damaged. For example, in Wuhan East Lake of China, the large amount of Grass carp were released to eliminate the water bloom and to increase the fishery production in 1970's. The result is that the plankton communities were damaged [1]. In order to avoid this situation and to maintain two species coexisting, it is

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necessary to carefully select the control measures and the relative control parameters (e.g., the amount of the released fish). In this paper, we will formulate a kind of impulsive semidynamical system and consider the feasibility and periodicity of impulsive state feedback control which depends on the biomass of the algae.

The bidirectional interactions of the algae and the fish are of consumer-resource(C-R) interactions. Holland and DeAngelis[2] developed a general theory for transitions between outcomes based on C-R interactions in which one or both species exploit the other as a resource. Simple models of C-R interactions indicated that the densities of the species alone could determine the fate of interactions. To test the influence of C-R interactions on the dynamics and stability of bi- and uni-directional C-R mutualisms, the simple models which link consumer functional response of one mutualistic species with the resources supplied by another is developed in paper [3]. Wang et al. [4] considered a predator-prey system of two species in which the predator consumes the prey and the prey has a harmful effect on the predator. By phase-portrait analysis and numerical simulations, it is demonstrated in paper [4] that interaction outcomes in the system may transition among predation, amensalism, competition, neutralism and commensalism. Varying initial densities of species population alone and varying one or more parameters (factors) can lead to the transition. Pal et al. [5] studied the effect of nutrient concentration and rate of toxin released by phytoplankton for the occurrence and termination of the planktonic bloom. Upadhyay et al. [6] investigated the dynamical complexities in two types of chaotic tri-trophic aquatic food-chain systems, where phytoplankton produce chemical substances known as toxins to reduce grazing pressure by zooplankton. There still are some references to investigate the C-R models, one can read the above papers and the references therein. Most of those models given in the above references considered the continuous models which have no the terms of impulsive state feedback control. We will consider an algae-fish system with impulsive state feedback control in which the algae as a food resource has negative effect on its consumer.

The ordinary differential equation with impulsive state effects is called as impulsive semidynamical system in Ref. [7] and semicontinous dynamical system in paper [8]. In those impulsive system, the conditions of impulsive effects depend on the state of the variables. The researchers have applied the impulsive semidynamical system on the biological mathematics fields, such as population system, turbidostat system and chemstat system. For examples, Tang and Cheke [9] proposed a state-dependent impulsive model for integrated pest management (IPM) and proved that there is no periodic solution with order larger than or equal to three, except for one special case, by using the properties of LambertW function and Poincaré map. Moreover, it is showed that the existence of an order two periodic solution implies the existence of an order one periodic solution [9].

Jiang and Lu [10] and Nie et al. [11] formulated and investigated the predator-prey models with impulsive state feedback control. The sufficient conditions for the existence and stability of semi-trivial solution and positive period-1 solution are obtained by using the Poincaré map and the analogue of the Poincaré criterion. Zeng et al. [12] generalized the Poincaré-Bendixson theorem of ordinary differential equation and gave an existence theorem of periodic solution of order one for a general planar autonomous impulsive system. Based on the ideas given in paper [12], some turbidostat systems and chemostat systems with impulsive state feedback control were proposed to investigate the periodicity of microorganism culture (e.g., [13–16]). Subsequently, Chen [8] gave the general ideas and methods such as the successor function to study the planar autonomous impulsive system. By using the successor function method, some mathematical models with impulsive state feedback control were formulated and investigated(e.g., [17–20]). The models in those papers have either the first integral or the stable equilibrium, or the limit cycle. But a few papers considered the impulsive semidynamical system with bistable property in which the positive equilibrium is a saddle point.

This paper will propose a kind of impulsive semidynamical system with bistable property to describe the evolution process of the algae and the fish under impulsive state feedback control, try to consider the feasibility and periodicity of impulsive state feedback control by investigating the existence and stability of periodic solution.

The rest of this paper is organized as follows. In Section 2, we will introduce a kind of simple consumer-resource system which can be viewed as a Kolmogorov-type system, and analyze its dynamical properties. For the bistable case, an impulsive semidynamical system is formulated. Some definitions and lemmas, the Poincaré map and the analogue of Poincaré criterion are also given in Section 2. The case of single releasing fish is discussed in Section 3 and the case of single chemical control is investigated in Section 4. The integrate control combining releasing fish and spraying algaecide is discussed in Section 5. Section 6 gives the numerical simulations and discussions.

2. Model formulation and preliminaries

2.1. Basic model

Since the fish consumes the algae and the algae has the negative effect on the fish, then we can use the consumer-resource(C-R) interaction model [2,3] to describe the bidirectional interactions of the algae and the fish. One of the bidirectional C-R interaction models can be written as the following form [21].

$$\begin{cases} \dfrac{dx_1}{dt} = x_1(r_1 + f_1(R_1(x_1, y_1)) - g_1(R_2(x_1, y_1)) - d_1x_1), \\ \dfrac{dy_1}{dt} = y_1(r_2 - g_2(R_1(x_1, y_1)) - d_2y_1), \end{cases} \tag{2.1}$$

where $x_1 = x_1(t)$ is assumed to be the consumer and $y_1 = y_1(t)$ the resource. The consumer x_1 feeds on the resource y_1. The resource y_1 has negative effect on the consumer x_1. The ratios r_1/d_1 and r_2/d_2 can be thought of as the carrying capacities in

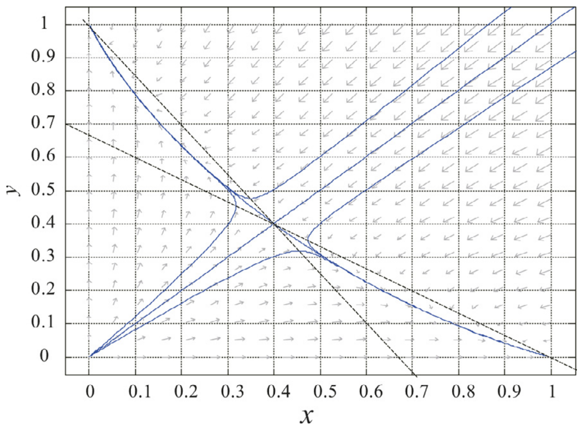

Fig. 1. The vector diagram of system (2.4) for $a_{12} > 1, a_{21} > 1$.

the absence of the other species. The term $f_1(R_1(x_1, y_1))$ represents its gain from the interaction, and $g_i(R_j(x_1, y_1))$ $(i, j = 1, 2)$ represents the costs incurred to it by the interaction [21]. $R_1(x_1, y_1)$ represents the interactions of predation and $R_2(x_1, y_1)$ the negative effects.

Since the functions $f_1(R_1(x_1, y_1))$, $g_1(R_2(x_1, y_1))$ and $g_2(R_1(x_1, y_1))$ have different possible forms (see Refs. [21,22]), and different combinations of the functions result in different dynamical complexities. In order to reveal clearly the effects of impulsive state feedback control on the system with bistable property, we choose $k_i = r_i/d_i (i = 1, 2), f_1(R_1(x_1, y_1)) = c_1 g_1(R_2(x_1, y_1)),$ $g_1(R_2(x_1, y_1)) = \frac{b_{12}}{r_1} y_1$ and $g_2(R_1(x_1, y_1)) = \frac{b_{21}}{r_2} x_1$, then system (2.1) has the following form.

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 x_1 \left(1 - \dfrac{x_1}{k_1} + c_1 b_{21} y_1 - b_{12} y_1\right), \\ \dfrac{dy_1}{dt} = r_2 y_1 \left(1 - \dfrac{y_1}{k_2} - b_{21} x_1\right), \end{cases} \tag{2.2}$$

where $x_1 = x_1(t)$ and $y_1 = y_1(t)$ are assumed to be the biomass of the fish (the consumer) and the algae (the resource) in a fresh water body (e.g., a reservoir). $r_1, r_2, k_1, k_2, b_{12}$ and b_{21} are the positive constants. r_1 and r_2 are the intrinsic growth rates of the fish and the algae in the absence of the other species, which implies that one species can maintain itself without the other one; k_1 and k_2 are the corresponding environment carrying capacities; c_1 is the conversion rate from the algae to the fish; b_{21} is the loss rate of the algae owing to the predation of the fish; b_{12} is the reduce rate of growth owing to the negative effect of the algae on the fish.

System (2.2) is of the classical Kolmogorov model, it can be viewed as a competitive model if $b_{12} - c_1 b_{21} > 0$, a predator-prey model if $b_{12} - cb_{21} < 0$.

In order to discuss the effects of impulsive state feedback on the bistable system, we assume that $b_{12} - cb_{21} > 0$ and let

$$x = \frac{x_1}{k_1}, \quad y = \frac{y_1}{k_2}, \quad t = r_1 t_1, \quad b = \frac{r_2}{r_1}, \quad a_{12} = (b_{12} - cb_{21})k_2, \quad a_{21} = b_{21} k_1, \tag{2.3}$$

then system (2.2) becomes

$$\begin{cases} \dfrac{dx}{dt} = x(1 - x - a_{12} y), \\ \dfrac{dy}{dt} = by(1 - y - a_{21} x). \end{cases} \tag{2.4}$$

It is easily obtained that system (2.4) has four equilibria: (0, 0), (0, 1), (0, 1) and (x^*, y^*) where

$$x^* = \frac{1 - a_{12}}{1 - a_{12} a_{21}}, \quad y^* = \frac{1 - a_{21}}{1 - a_{12} a_{21}}.$$

The equilibrium (x^*, y^*) is a saddle point and system (2.4) is bistable for $a_{12} > 1, a_{21} > 1$(see Ref.[23]). In this case, only one of two species can survive and the other one tends to be extinct, which is determined by their initial values. The illustration of vector diagram of system (2.4) for $a_{12} > 1, a_{21} > 1$ can be seen in Fig. 1.

Since the equilibrium (x^*, y^*) is a saddle point for $a_{12} > 1, a_{21} > 1$, then system (2.4) has four saddle point separatrixes(denoted by s_1, s_2, s_3, s_4, respectively) which divide the first quadrant into four regions (see Fig. 2), denoted by Q_1, Q_2, R_1, R_2, respectively, and let $Q = Q_1 + Q_2, R = R_1 + R_2$.

It is easily known that the trajectories tend to the equilibrium (0, 1) and the fish species x tends to be extinct if the initial point lies in the region Q, the trajectories tend to the equilibrium (1, 0) and the algae species y tends to be extinct if the initial point lies in the region R. From the perspective of protecting the diversity of species in a habitat, the above two cases are not what we want

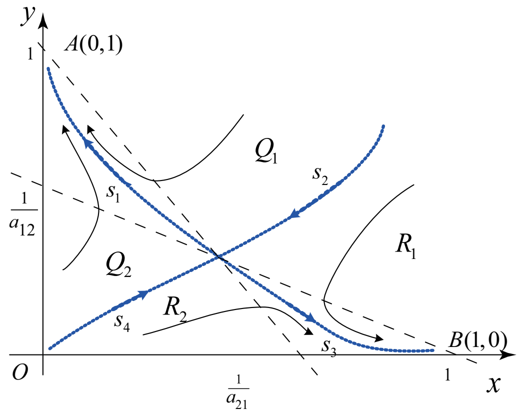

Fig. 2. The saddle point separatrixes of system (2.4) for $a_{12} > 1$ and $a_{21} > 1$.

to see. We hope that some control strategies are taken to prevent them from extinction and to maintain the biomass of them in a suitable region.

2.2. Model formulation

We want to consider the feasibility of controlling the excessive growth of the algae and preventing the fish from extinction by using the impulsive state feedback control. The researchers have presented and designed the early warning systems and the monitoring systems to monitor the biomass of the algal and other factors which can cause the algal blooms [24,25]. For example, Cheng [25] has investigated and presented the early warning system of algal development in Lake Dianshan, China. The first alert level (early warning) and the second alert level (bloom warning) of the algal were presented in Ref. [25]. When the biomass of the algal reaches the first alert level or the second alert level, some measures should be taken to control the algal. The control measures to the algae mainly have chemical method (e.g., spraying algaecide), mechanical method, biological and ecological method (e.g., releasing fish), flocculation method (using flocculant) and so on. We mainly consider the following measures:

1. Spraying algaecide. By using this control measure, the amount of the algae is decreased proportionally, and the increment of the algae in system (2.2) can be written as $\Delta y_1 = y_1^+ - y_1 = -\beta_1 y_1$ where y_1^+ is the population of the algae after spraying algaecide, $0 \le \beta_1 < 1$.
2. Releasing fish. By the activity of releasing fish, the biomass of the fish is added. Usually, the amount of the released fish is constant and the increment of the fish in system (2.2) can be written as $\Delta x_1 = x_1^+ - x_1 = p_1$ where x_1^+ is the biomass of the fish after releasing fish. $p_1 \ge 0$ is a constant.

Our aim is to maintain the biomass of the fish and the algae in a suitable region where the biomass of the algae is less than the threshold value, therefore we will discuss system (2.4) in the region $Q = Q_1 + Q_2$. In the region Q, if no control measure is taken, then the algae reaches its environmental carrying capacity and the fish tends to be extinct. Suppose that the above control measures are taken when the biomass of the algae reaches a threshold value denoted by h_1(e.g., the first alert level or the second alert level given in Ref. [25]). By using the same dimensionless transformation (2.3) and let $h = \frac{h_1}{k_1}$, $\beta = \beta_1, p = \frac{p_1}{k_2}$, then system (2.2) incorporating the impulsive state feedback control has the following form:

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= x(1 - x - a_{12}y), \\ \frac{dy}{dt} &= by(1 - y - a_{21}x), \end{aligned}\right\} & y < h, \\ \Delta x = p, & \\ \Delta y = -\beta y, & y = h, \\ y(0) < h. & \end{cases} \tag{2.5}$$

where $p \ge 0$, $0 \le \beta < 1$, $\Delta x = x^+ - x$, $\Delta y = y^+ - y$, and

$$x^+ = x(t_k^+) = \lim_{h \to 0^+} x(t_k + h), \quad y^+ = y(t_k^+) = \lim_{h \to 0^+} y(t_k + h).$$

The feasibility and periodicity of the impulsive state feedback control can be given by discussing the existence and stability of order-1 periodic solution of system (2.5). For system (2.5), if $\beta = 0$ or $p = 0$, then it implies that only one control is taken. That is, only the activity of releasing fish is performed if $\beta = 0$ and $p \neq 0$. Similarly, only the measure of spraying algaecide is taken if $p = 0$ and $\beta \neq 0$.

We will discuss the existence and stability of order-1 periodic solution of system (2.5) for three cases: (1) $\beta = 0, p > 0$, (2) $p = 0, 0 < \beta < 1$ and (3) $0 < \beta < 1, p > 0$, respectively.

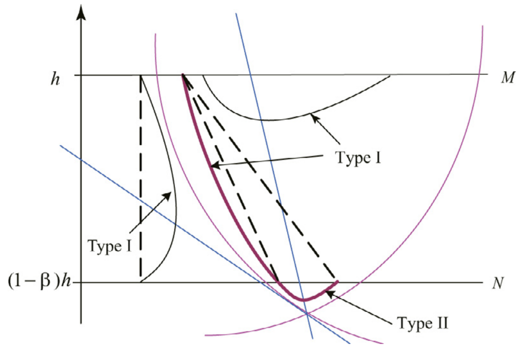

Fig. 3. The illustration of two kinds of order-1 periodic cycle of system (2.5).

2.3. Preliminaries

System (2.5) is of impulsive semidynamical system. The definitions of impulsive semidynamical system can be found in Ref. [7]. Here only gives some definition and notations. More details can be seen in Ref. [7].

Definition 2.1 (Lakshmikantham, et al. [7]). A trajectory $\tilde{\pi}_x$ is said to be periodic of order k if there exist positive integers $m \geq 1$ and $k \geq 1$ such that k is the smallest integer for which $x_m^+ = x_{m+k}^+$.

The impulsive set of system (2.5) is $M = \{(x, y)|y = h\}$ and its image set is $N = \{(x, y)|y = (1 - \beta)h\}$. Without distinction, we sometimes call the line $y = h$ the impulsive set and the line $y = (1 - \beta)h$ the image set.

For any point $P \in \{(x, y)|x > 0, y > 0\}$, we give the following notations for simplicity.

- $\pi_P(t) = \pi(P, t)$: the trajectory starting from the point P.
- $\pi^+(P) = \{\pi(P, t)|0 \leq t < +\infty\}$: the positive semi-trajectory starting from the point P.
- $\pi^-(P) = \{\pi(P, t)| - \infty < t \leq 0\}$: the negative semi-trajectory starting from the point P.
- $\pi_1(P)$: the first intersection point of $\pi^+(P)$ and M, that is, there exists a $t_1 \in \mathbf{R}^+$ such that $\pi_1(P) = \pi(P, t_1) \in M$, and for $0 < t < t_1, \pi(P, t) \notin M$.
- x_P: the abscissa of the point P and y_P the ordinate.
- P^+: If $P \in M$, then the impulsive effect occurs at the point P, the impulsive functions $\psi(x, y) = (\Delta x, \Delta y)$ transfers the point P into P^+.

Definition 2.2 [8,26]. A trajectory $\tilde{\pi}(P, t)$ is called order-1 periodic solution with period T if there exist a point $P \in N$ and $T > 0$ such that $\pi(P, T) = Q \in M$ and $\psi(Q) = \psi(\pi(P, T)) = P \in N$. The trajectory $\tilde{\pi}(P, t)$ linking with the impulsive line segment QP is also called an order-1 cycle. If the order-1 cycle has a singularity, then it is called an order-1 singular cycle.

Definition 2.3. Assume that system (2.5) has an order-1 periodic solution $\tilde{\pi}(A, t)$ with period T and $A \in N$, if $\tilde{\pi}(A, t)$ is monotonous with respect to a variable (x or y) and $\tilde{\pi}(A, t) \cap N = \varnothing, t \in (0, T)$, then $\tilde{\pi}(A, t)$ is called as the order-1 periodic solution of type I, denoted by Γ_1. If $\tilde{\pi}(A, t)$ is not monotonous and $\tilde{\pi}(A, t) \cap N \neq \varnothing, t \in (0, T)$. That is, if the number of the intersection point of $y = (1 - \beta)h$ and $\tilde{\pi}(A, t), t \in (0, T)$ is one, then $\tilde{\pi}(A, t)$ is called as the order-1 periodic solution of type II, denoted by Γ_2 (see Fig. 3).

In order to investigate the qualitative properties of system (2.5), we need introduce the definitions of successor point and successor function.

Definition 2.4 [8,18]. Let M and N be the lines where the impulsive set and its image set lie on, respectively (see Fig. 4(a)). Define a new coordinate axis O' on the line N, the direction and length unit of the new coordinate axis are the same as those of the axis-x. For any point $A(x, y) \in N, x > 0, y > 0$, the new coordinate of $A(x, y)$ denotes by $l(A)$ and $l(A) = x$.

For any point $A(x_0, y_0) \in N$, the trajectory $\pi(A, t)$ of system (2.5) hits the impulse set M, and then jumps to $A_1(x_1, y_1) \in N$, where $y_0 = y_1 = (1 - p)h$, then the point A_1 is said to the success point of A, and the success function can be written as $f(A) = l(A_1) - l(A) = x_1 - x_0$.

According to the continuity of compound function, we know that the successor function $f(A)$ is continuous. Obviously, if $f(A) = 0$, then the trajectory $\pi(A, t)$ is an order-1 periodic solution $\tilde{\pi}(A, t)$ of system (2.5).

2.4. Poincaré map

According to the ideas of Ref. [10], the followings establish a kind of Poincaré maps to discuss the stability of periodic solution of system (2.5). Firstly, let $S_1 = \{(x, y)|y = h, x \geq 0\}$ be the Poincaré section. Suppose system (2.5) has an order-1 periodic solution $(\phi(t), \varphi(t))$ with period T (see Fig. 4(b)). The periodic trajectory with initial point $E^+(x_0, (1 - \beta)h)$ intersects the Poincaré section

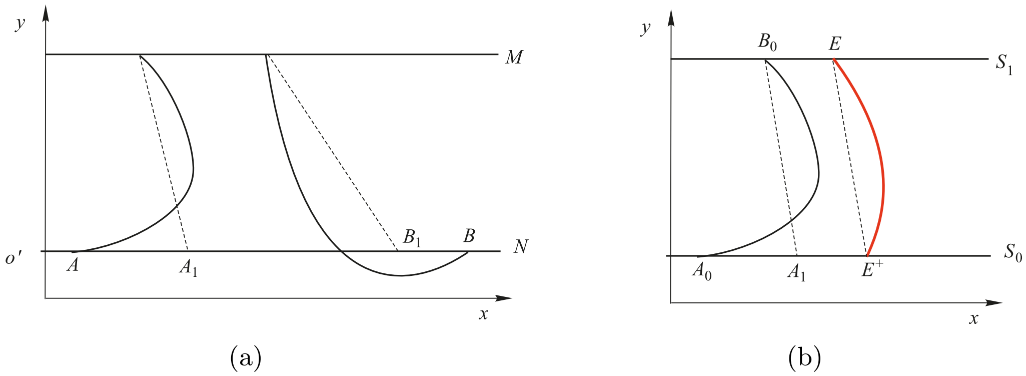

Fig. 4. The illustration of the success function and the Poincaré map of system (2.5).

S_1 at the point $E(x_1, h)$, then jumps to the point E^+ on the line $S_0 = \{(x, y)|y = (1-\beta)h, x \geq 0\}$ under the impulsive effects $\Delta x = p$ and $\Delta y = (1-\beta)h$. Therefore,

$$\phi(0) = x_0, \quad \varphi(0) = (1-\beta)h, \quad \phi(T) = x_1, \quad \varphi(T) = h.$$

Consider another solution $(\overline{\phi}(t), \overline{\varphi}(t))$ with initial point $A_0(x_0 + \delta x_0, (1-\beta)h)$, where δx_0 is small enough. This disturbed trajectory $\pi^+(A_0)$ first intersects the Poincaré section S_1 at the point $B_0(\overline{x}_1, h)$ at the moment $t = T + \delta t$ and then jumps to the point $A_1(\bar{x}, (1-\beta)h)$ on the line S_0. Hence,

$$\overline{\phi}(0) = x_0 + \delta x_0, \quad \overline{\varphi}(0) = (1-\beta)h, \quad \overline{\phi}(T + \delta t) = \overline{x}_1, \quad \overline{\varphi}(T + \delta t) = h.$$

Let $\delta x = \overline{\phi}(t) - \phi(t)$ and $\delta y = \overline{\varphi}(t) - \varphi(t)$, then $\delta x_0 = \overline{\phi}(0) - \phi(0) = |A_0E^+|$ and $\delta y_0 = \overline{\varphi}(0) - \varphi(0) = 0$. Set $\delta x_1 = |A_1E^+|$ and $\delta x_0^* = |B_0E|$, and then the relation between δx_0 and δx_0^* determines one type of Poincaré map. For $0 < t < T$, δx and δy are described by the following equation.

$$\begin{pmatrix} \delta x(t) \\ \delta y(t) \end{pmatrix} = M(t)\begin{pmatrix} \delta x_0 \\ \delta y_0 \end{pmatrix} + o(\delta x_0^2 + \delta y_0^2) = M(t)\begin{pmatrix} \delta x_0 \\ 0 \end{pmatrix} + o\begin{pmatrix} \delta x_0^2 \\ 0 \end{pmatrix}, \tag{2.6}$$

where the fundamental solution matrix $M(t)$ satisfies the variational equation

$$\frac{\mathrm{d}M(t)}{\mathrm{d}t} = V(t)M(t), \quad M(0) = I_2, \tag{2.7}$$

where I_2 is the unit matrix of second order, the elements of $V(t)$ can be calculated along the periodic trajectory $(\phi(t), \varphi(t))$ and

$$V(t) = \begin{pmatrix} 1 - 2\phi(t) - a_{12}\varphi(t) & -a_{12}\phi(t) \\ -ba_{21}\varphi(t) & b(1 - 2\varphi(t) - a_{21}\phi(t)) \end{pmatrix}. \tag{2.8}$$

Let $f_1(t) = \phi(t)(1 - \phi(t) - a_{12}\varphi(t))$, $f_2(t) = b\varphi(t)(1 - \varphi(t) - a_{21}\phi(t))$. For $t = T + \delta t$, the disturbed trajectory $(\overline{\phi}(t), \overline{\varphi}(t))$ is expressed in the following first-order Taylor expansion:

$$\begin{cases} \overline{\phi}(T + \delta t) \approx \phi(T) + \delta x(T) + f_1(T)\delta t, \\ \overline{\varphi}(T + \delta t) \approx \varphi(T) + \delta y(T) + f_2(T)\delta t. \end{cases} \tag{2.9}$$

It follows from $\overline{\varphi}(T + \delta t) = h$ and $\varphi(T) = h$ that we have

$$\delta t = -\frac{\delta y(T)}{f_2(T)}$$

and

$$\delta x_0^* = |B_0E| = \overline{x}_1 - x_1 = \overline{\phi}(T + \delta t) - \phi(T) = \delta x(T) - \frac{f_1(T)\delta y(T)}{f_2(T)}.$$

From $\bar{x} = \bar{x}_1 + p$, we have $\bar{x} - x_0 = \bar{x}_1 + p - (x_1 + p) = \bar{x}_1 - x_1$, that is, $\delta x_1 = \delta x_0^*$. The Poincaré map is constructed as

$$\delta x_1 = \delta x(T) - \frac{f_1(T)\delta y(T)}{f_2(T)}, \tag{2.10}$$

where $\delta x(T)$ and $\delta y(T)$ are calculated according to (2.6).

Lemma 2.5 (*Analogue of the Poincaré criterion* [27]). *The T-periodic solution $x = \xi(t), y = \eta(t)$ of the system*

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = P(x, y), \ \dfrac{\mathrm{d}y}{\mathrm{d}t} = Q(x, y), & if\ \phi(x, y) \neq 0, \\ \Delta x = A(x, y), \Delta y = B(x, y), & if\ \phi(x, y) = 0 \end{cases}$$

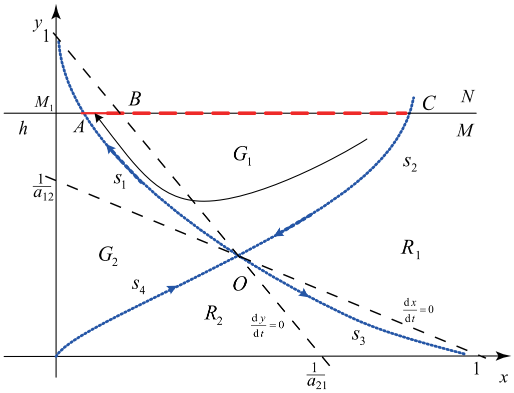

Fig. 5. The existence of order-1 singular cycle for $\beta = 0$, $p > 0$ and $h > y^*$.

is orbitally asymptotically stable and enjoys the property of asymptotic phase if the multiplier μ_2 *satisfies the condition* $|\mu_2| < 1$, *where*

$$\mu_2 = \prod_{k=1}^{q} \Delta_k \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t))\right) \mathrm{d}t\right],$$

$$\Delta_k = \frac{P_+\left(\frac{\partial B}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial B}{\partial x}\frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial x}\right) + Q_+\left(\frac{\partial A}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right)}{P\frac{\partial \phi}{\partial x} + Q\frac{\partial \phi}{\partial y}}$$

and $P, Q, \frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial B}{\partial x}, \frac{\partial B}{\partial y}, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}$ *are calculated at the point* $(\xi(\tau_k), \eta(\tau_k))$ *and* $P_+ = P(\xi(\tau_k^+), \eta(\tau_k^+))$, $Q_+ = Q(\xi(\tau_k^+), \eta(\tau_k^+))$.

If $h < y^*$, since $\frac{\mathrm{d}x}{\mathrm{d}t} > 0$ for the points in the region where $x < 1$ and $y < 1$, then we have the following theorem and omit the proof.

Theorem 2.6. *If* $h < y^*$, *then the trajectories starting from the region where* $x < 1$ *and* $y < h$ *will tend to (1,0) after at most finite times impulsive effects.*

Therefore, the followings always assume that $h > y^*$. Assume that the line $y = h$ intersects the axis-y, two separatrixes s_1, s_2 and the isoclinal line $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ at the points M_1, A, C, B, respectively (see Fig. 5), let $A = A(x_A, h)$, $B = B(x_B, h)$, $C = C(x_C, h)$, $p_1^* = x_C - x_A$, $p_2^* = x_C - x_B$ where $x_B = \frac{1-h}{a_{21}}$.

Denote the region we consider by G and $G = G_1 + G_2$ where $G_1 \in Q_1$ be the region whose boundary consists of the line $y = h$, the separatrixes s_1 and s_2; $G_2 \in Q_2$ be the region whose boundary consists of axis-y, the line $y = h$ and the separatrixes s_4 and s_1 (see Fig. 5).

We want to know whether the trajectories starting from the region G can stay there for a given value of p or β.

3. Order-1 periodic solution for $\beta = 0$ and $p > 0$

If $\beta = 0$ and $p > 0$, that is, only the activity of releasing fish is performed, then the impulsive functions of system (2.5) are $\Delta x = p$ and $\Delta y = 0$.

Theorem 3.1. *If* $a_{12} > 1$, $a_{21} > 1$, $h > y^*$, $\beta = 0$ *and* $p > 0$, *then there is a value* p_1^* *such that system (2.5) has an unstable order-1 singular cycle for* $p = p_1^*$.

Proof. From the qualitative properties of system (2.5) without the impulsive effects, we know that all the trajectories starting from the points in the region $G = G_1 + G_2$ and $y^* < y(0) < h$ hit the impulsive set M at the line segment $\overline{M_1B}$ where $0 \le x \le x_B$ and $y = h$.

If $\beta = 0$, since the impulsive set M and its image set N coincide (see Fig. 5), then it is obvious that there is a value $p_1^* = x_C - x_A$ such that the order-1 singular cycle $OACO$ exists and consists of the arc $\widehat{OA}$, the line segment $\overline{AC}$,the arc $\widehat{CO}$ and the equilibrium O, where the arcs $\widehat{OA}$ and $\widehat{CO}$ are on the separatrixes s_1 and s_2, respectively.

If $p = p_1^*$, then the trajectories starting from the region G_2 will go into the region G_1 after several impulsive effects. The trajectories starting from G_1 will hit the impulsive set M at the right neighborhood of point A on the line segment $\overline{M_1B}$, and jump to the region R_1, subsequently tend to the equilibrium $(1, 0)$. Since the trajectories cannot go back to region G after one impulsive effect, it is obvious that the singular cycle $OACO$ is unstable. This completes the proof. □

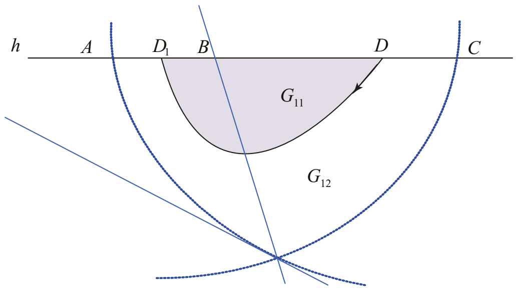

Fig. 6. The illustration of G_{11} and G_{12} for $p_2^* \le p < p_1^*$, $\beta = 0$ and $h > y^*$.

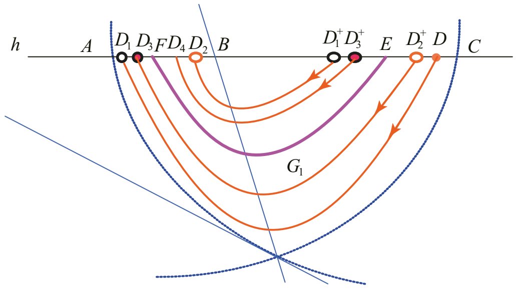

Fig. 7. The existence of order-1 periodic solution of system (2.5) for $p < p_2^*$, $\beta = 0$ and $h > y^*$.

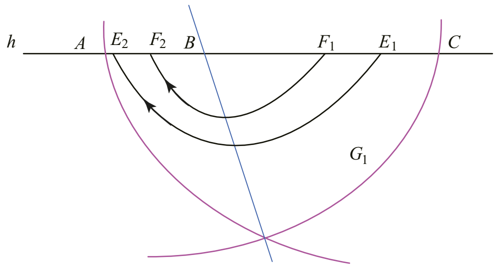

Fig. 8. The illustration of the uniqueness of order-1 periodic solution for $p < p_2^*$, $\beta = 0$ and $h > y^*$.

From Theorem 3.1, we know that if $p \ge p_1^* = x_C - x_A$, then the trajectories starting from the region G will go into the region R_1 after several impulsive effects, and tend to the equilibrium $(1, 0)$. This implies that the initial state and the dominant position of the species can be changed if the larger amount of the fish are released.

If $p_1^* > p \ge p_2^* = x_C - x_B$, then there is a point D_1 such that $|CD_1| = p > p_2^*$, the trajectory $\pi^-(D_1)$ intersecting the line segment $\overline{BC}$ at a point D. Further, the region G_1 can be divided into two regions G_{11} and G_{12} by the trajectory $\pi^+(D)$ between the points $\pi_1(D)$ and D, where the boundary of the region G_{11} consists of the line segment $\overline{DD_1}$ and the trajectory of $\pi^+(D)$(see Fig. 6). The trajectories starting from the region G_{11} will go into the region R_1 and the trajectories starting from the region G_{12} retain here. Therefore, in the following, we will consider the existence and stability of order-1 periodic solution for $p < p_2^*$ in the region G.

Theorem 3.2. *System (2.5) has an unique orbitally asymptotically stable order-1 periodic solution for $a_{12} > 1$, $a_{21} > 1$, $h > y^*$ and $\beta = 0$ if $p < p_2^*$ where $p_2^* = x_C - x_B$.*

Proof. If $p < p_2^*$, then there is a point D in the line segment $\overline{BC}$ satisfying $|BD| = p$. The trajectory $\pi^+(D)$ starting from the point D hits the impulsive set at the point D_1 and is mapped to the point D_1^+, then we have that $x_A < x_{D_1} < x_B$ and $x_B < x_{D_1^+} < x_D$. Since the point D_1^+ is the successor point of D, then the successor function $f(D) = x_{D_1^+} - x_D < 0$(see Fig. 7).

For the point D_1^+, the trajectory $\pi^+(D_1^+)$ hits the impulsive set at the point D_2 and then jumps to the point D_2^+ under the impulsive effect. The point D_2^+ is the successor point of D_1^+ and $x_{D_2^+} > x_{D_1^+}$ since $x_{D_2} > x_{D_1}$. Further, $f(D_1^+) = x_{D_2^+} - x_{D_1^+} > 0$. Therefore, there must be a point E between D_1^+ and D such that $f(E) = 0$ which implies that system (2.5) has an order-1 periodic solution $\widetilde{\pi}_E(t)$ starting from the point E.

For $p < p_2^*$, suppose that there are two order-1 periodic solutions $\widetilde{\pi}_{E_1}(t)$ and $\widetilde{\pi}_{F_1}(t)$ starting from the points E_1 and F_1, respectively(see Fig. 8). Without loss of generality, assume that two order-1 periodic solutions hit the impulsive set at the points E_2 and

F_2, respectively. The points E_2 and F_2 are on the line segment $\overline{AB}$. The points E_1 and E_2 are on the same trajectory $\widetilde{\pi}_{E_1}(t)$, the points F_1 and F_2 are on the same trajectory $\widetilde{\pi}_{F_1}(t)$. Since the trajectories of autonomous system cannot intersect, then it should be that $x_{E_2} < x_{F_2} < x_{F_1} < x_{E_1}$. But according to the impulsive function $\Delta x = p$, we know that $x_{E_1} = x^+_{E_2} = x_{E_2} + p < x_{F_2} + p = x^+_{F_2} = x_{F_1}$, which concludes a contradiction. Therefore, system (2.5) has an unique order-1 periodic solution for $p < p^*$.

The followings discuss the stability of the order-1 periodic solution. Denote the order-1 periodic solution by $\widetilde{\pi}_E(t)$ which starts from the point E and intersects the line segment $\overline{AB}$ at the point F. Consider the successor point D^+_1 of D(see Fig. 7), we know that $x_{D_1} < x_F < x_B$ and $x_B < x_{D^+_1} < x_E < x_D$. The trajectory $\pi^+(D^+_1)$ must intersect the impulsive set again at the point D_2 and then be mapped to the point D^+_2 which is the successor point of D^+_1. Because the trajectories cannot intersect, we can easily know that $x_F < x_{D_2} < x_B$ and $x_E < x_{D^+_2} < x_D$.

Similarly, the trajectory $\pi^+(D^+_2)$ must intersect the impulsive set again at the point D_3 and then be mapped to the point D^+_3 which is the successor point of D^+_2. We have that $x_{D_1} < x_{D_3} < x_F < x_{D_2} < x_B$ and $x_{D^+_1} < x_{D^+_3} < x_E < x_{D^+_2} < x_D$.

Repeating the above steps, the trajectory starting from the point D will undergo the impulsive effect infinitely times. Denote the image point corresponding to the ith impulsive effect by D^+_i, $i = 1, 2, \ldots$. It follows that

$$x_{D^+_1} < x_{D^+_3} < \cdots < x_{D^+_{2k-1}} < \cdots < x_E$$

and

$$x_{D^+_2} > x_{D^+_4} > \cdots > x_{D^+_{2k}} > \cdots > x_E.$$

Thus $\{x_{D^+_{2k-1}}\}, k = 1, 2, \ldots,$ is a monotonically increasing sequence, and $\{x_{D^+_{2k}}\}, k = 1, 2, \ldots,$ is a monotonically decreasing sequence (see Fig. 7), and furthermore, $x_{D^+_{2k+1}} \to x_E$ as $k \to \infty$; $x_{D^+_{2k}} \to x_E$ as $k \to \infty$.

Choose an arbitrary point P_0 in the line segment $\overline{D^+_1 D}$, which is different from the point E. Without loss of generality, we assume that $x_{D^+_1} < x_{P_0} < x_E$(otherwise, $x_E < x_{P_0} < x_D$ and the argument is similar). There must exist an integer k such that $x_{D^+_{2k-1}} < x_{P_0} < x_{D^+_{2k+1}}$, $k = 1, 2, \ldots$. The trajectory $\pi^+(P_0)$ will also undergo infinite times impulsive effects. Denote the image point corresponding to the lth impulsive effect by P^+_l, $l = 1, 2, \ldots,$ then for any l, $x_{D_{2k+l-1}} < x_{P_l} < x_{D_{2k+l+1}}$ and $x_{D^+_{2k+l-1}} < x_{P^+_l} < x_{D^+_{2k+l+1}}$, and thus the sequence $\{x_{P_{2l-1}}\}, l = 1, 2, \ldots,$ is monotonically increasing, and the sequence $\{x_{P^+_{2l}}\}, l = 1, 2, \ldots,$ is monotonically decreasing, further $x_{P^+_{2l+1}} \to x_E$ as $l \to \infty$; $x_{P^+_{2l}} \to x_E$ as $l \to \infty$.

Therefore, in either case, the successor points of the image points tend to the point E, which implies that the order-1 periodic solution of system (2.5) is orbitally asymptotically stable. This completes the proof. □

Remark 3.3. Assume that the arc $\widehat{EF}$ is the trajectory of the order-1 periodic solution for $p = p_1 < p^*_2$. With p decreasing, let $p = p_2 < p_1 < p^*_2$, then the order-1 successor point E_2 of E is on the left of the point E for $p = p_2 < p_1$. According to the ideas and proof of Theorem 3.2, system (2.5) also has an unique order-1 periodic solution for $p = p_2$. Besides, the points E and F are close to the point B with p decreasing. With p decreasing and $p \to 0$, the trajectory contracts along the direction to the point B.

Remark 3.4. Since the impulsive set coincides with its image set at the line $y = h$, if p is small enough, the trajectories starting from the region G_2 will hit the impulsive set M at a point and the point will jump continuously along the line $y = h$ until the point reaches the right side of the point B.

4. Order-1 periodic solution for $p = 0$ and $0 < \beta < 1$

If $p = 0$ and $0 < \beta < 1$, then the impulsive functions of system (2.5) are $\Delta x = 0$ and $\Delta y = -\beta y$. It is obvious that the trajectories starting from the region G_1 will go into the region G_2 after several impulsive effects. Therefore, we will mainly consider the existence of order-1 periodic solution in the region G_2.

Theorem 4.1. *If $h > y^*$, $a_{12} > 1$, $a_{21} > 1$, $p = 0$ and $0 < \beta < 1$, then there is a value β^* such that system (2.5) has an unstable order-1 singular cycle for $\beta = \beta^*$ and $(1 - \beta)h < y^*$.*

Proof. Since $(1 - \beta)h < y^*$, then we can assume that the image set $N = \{(x, y)|y = (1 - \beta)h\}$ intersects with the separatrix s_4 at the point E (see Fig. 9). Obviously, there exists a value β^* such that $x_A = x_E$ for $\beta = \beta^*$ where A is the intersection point of the impulsive set $M = \{(x, y)|y = h\}$ and the separatrix s_1. For $\beta = \beta^*$, the point near the equilibrium O along the separatrix s_1 reaches the point A, jumps to the point E and then tends to the equilibrium O along the separatrix s_4. Therefore, the order-1 singular cycle consists of the arc $\widehat{OA}$, the line segment $\overline{AE}$, the arc $\widehat{EO}$ and the equilibrium O for $p = 0$ and $\beta = \beta^*$. Similar to the proof of Theorem 3.2, it can also verify that the order-1 singular cycle is unstable. This completes the proof. □

Theorem 4.2. *If $a_{12} > 1$, $a_{21} > 1$, $p = 0$ and $0 < \beta < 1$, then system (2.5) has an boundary order-1 periodic solution.*

If $(1 - \beta)h \geq \frac{1}{a_{12}}$, then the boundary order-1 periodic solution is stable.

If $(1 - \beta)h < \frac{1}{a_{12}}$, then the boundary order-1 periodic solution is unstable.

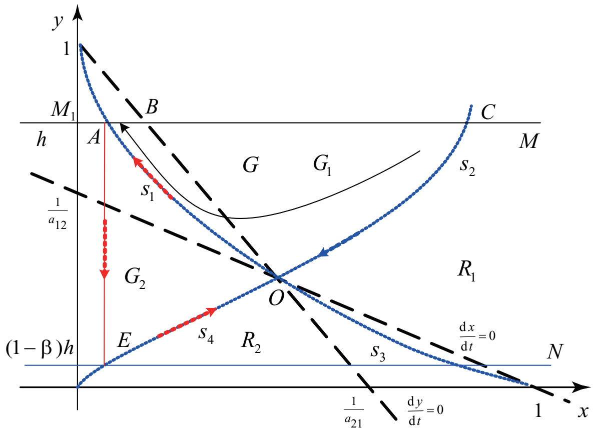

Fig. 9. The existence of order-1 singular cycle of system (2.5) for $p = 0$ and $h > y^*$.

Proof. To discuss the existence of the boundary periodic solution, let $x = 0$, then system (2.5) becomes

$$\begin{cases} \frac{\mathrm{d}y}{\mathrm{d}t} = by(1-y), & y < h, \\ \Delta y = -\beta y, & y = h, \\ y(0) < h. \end{cases} \tag{4.1}$$

For $t \in (t_{k-1}, t_k]$ and $y(t_k) = h, k = 1, 2, \ldots,$ it follows that

$$y(t) = \frac{y_k^+}{y_k^+ - (y_k^+ - 1)\mathrm{e}^{-b(t-t_k)}}, t \in (t_{k-1}, t_k],$$

where $y_k^+ = y(t_k^+)$ and t_k^+ is the initial impulsive moment for $t \in (t_{k-1}, t_k], k = 1, 2, \ldots$. Since $y(t_k) = h$ and $y_k^+ = (1-\beta)h$, then we can obtain the boundary periodic solution of system (4.1) as follows:

$$\widetilde{y}(t) = \frac{(1-\beta)h}{(1-\beta)h - ((1-\beta)h - 1)\mathrm{e}^{-b(t-kT)}},$$

where T is the period of the periodic solution $\widetilde{y}(t)$, that is, $t_k = t_{k-1} + T, k = 1, 2, \ldots$ and

$$T = \frac{1}{b} \ln \frac{(1-\beta)h - 1}{(1-\beta)(h-1)}.$$

Therefore system (2.5) has a boundary periodic solution $(0, \widetilde{y}(t))$. The followings discuss the stability of $(0, \widetilde{y}(t))$ by using the Poincaré map. From Eq. (2.7), it follows that $\phi(t) = 0, \varphi(t) = \widetilde{y}(t)$ and

$$\frac{\mathrm{d}M(t)}{\mathrm{d}t} = \begin{pmatrix} 1 - a_{12}\widetilde{y}(t) & 0 \\ -ba_{12}\widetilde{y}(t) & b(1 - 2\widetilde{y}(t)) \end{pmatrix} M(t), \quad M(0) = I_2. \tag{4.2}$$

Let

$$M(t) = \begin{pmatrix} m(t) & n(t) \\ u(t) & v(t) \end{pmatrix},$$

then Eq. (4.2) can be rewritten as the following form for $0 < t < T$,

$$\begin{cases} \frac{\mathrm{d}m(t)}{\mathrm{d}t} = (1 - a_{12}\widetilde{y}(t))m(t), & m(0) = 1, \\ \frac{\mathrm{d}n(t)}{\mathrm{d}t} = (1 - a_{12}\widetilde{y}(t))n(t), & n(0) = 0, \\ \frac{\mathrm{d}u(t)}{\mathrm{d}t} = (-ba_{12}\widetilde{y}(t))m(t) + b(1 - 2\widetilde{y}(t))u(t), & u(0) = 1, \\ \frac{\mathrm{d}u(t)}{\mathrm{d}t} = (-ba_{12}\widetilde{y}(t))n(t) + b(1 - 2\widetilde{y}(t))v(t), & v(0) = 0. \end{cases} \tag{4.3}$$

Since $\delta y_0 = 0$ and $f_1(T) = 0 \times (1 - 0 - a_{12}\varphi(t))$, it is only need to calculate $m(t)$. From the first equation of Eq.(4.3), we have

$$m(t) = \exp\left(t - a_{12}t - \frac{a_{12}}{b} \ln((1-\beta)h - ((1-\beta)h - 1)\mathrm{e}^{-bt})\right). \tag{4.4}$$

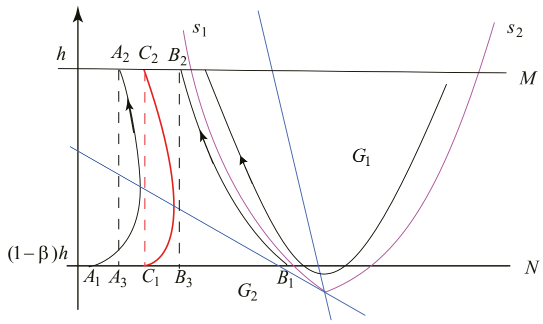

Fig. 10. The existence of order-1 periodic solution for $p = 0$, $h > \frac{1}{a_{12}} > (1-\beta)h > y^*$ and $\beta_* < \beta < \beta^*$.

In view of Eq. (2.6), we have

$$\delta x(T) = m(T)\delta x_0,$$

where

$$m(T) = \exp\left(T - a_{12}T - \frac{a_{12}}{b}\ln\left((1-\beta)h - ((1-\beta)h - 1)e^{-bT}\right)\right). \tag{4.5}$$

Obviously, $\delta x_0 = 0$ is a fixed point of $\delta x(T) = m(T)\delta x_0$.

Consider the expression of the period T, let

$$g(\beta) = \frac{1}{1-\beta}\left(\frac{1-h}{1-(1-\beta)h}\right)^{a_{12}-1}.$$

If $g(\beta) < 1$ holds, then $0 < m(T) < 1$. Furthermore, $\delta x_0 = 0$ is a stable fixed point and system (2.5) has a stable boundary order-1 periodic solution.

Since $\beta \in (0, 1)$, then we have that $g(0) = 1$ and $g(1) = +\infty$. From

$$\frac{\partial g(\beta)}{\partial \beta} = (1-h)^{a_{12}-1}(1-(1-\beta)h)^{a_{12}-2}(1-(1-\beta)ha_{12}),$$

we know that $\frac{\partial g(\beta)}{\partial \beta} = 0$ if $1-(1-\beta)ha_{12} = 0$. $\frac{\partial g(\beta)}{\partial \beta} < 0$ and $g(\beta) < 1$ if $1-(1-\beta)ha_{12} < 0$ (that is $(1-\beta)h > \frac{1}{a_{12}}$). $\frac{\partial g(\beta)}{\partial \beta} > 0$ and $g(\beta) > 1$ if $1-(1-\beta)ha_{12} > 0$. Therefore, the boundary periodic solution $(0, \tilde{y}(t))$ of system (2.5) is stable if $(1-\beta)h \ge \frac{1}{a_{12}}$, unstable if $(1-\beta)h < \frac{1}{a_{12}}$. This completes the proof. □

Remark 4.3. Since $(1-\beta)h < \frac{1}{a_{12}}$ must hold if $h < \frac{1}{a_{12}}$, then the boundary order-1 periodic solution $(0, \tilde{y}(t))$ of system (2.5) must be unstable for $h < \frac{1}{a_{12}}$.

From Theorem 4.1, we know that if $\beta > \beta^*$, then all the trajectories starting from the region G_2 will enter the region R_2 after several impulsive effects and tends to the equilibrium (1,0). Therefore, the followings will consider the existence of order-1 periodic solution for $\beta_* < \beta < \beta^*$, where $\beta_* = 1 - \frac{1}{a_{12}h}$.

For $\beta_* < \beta < \beta^*$, the positions of the impulsive set M and its image set N have the following cases:

(1) $h > \frac{1}{a_{12}} > (1-\beta)h > y^*$;
(2) $h > \frac{1}{a_{12}}, (1-\beta)h < y^*$;
(3) $h < \frac{1}{a_{12}}, (1-\beta)h > y^*$;
(4) $h < \frac{1}{a_{12}}, (1-\beta)h < y^*$.

Because the proofs of the existence of order-1 periodic solution for above four cases are similar, we only consider the case of $h > \frac{1}{a_{12}} > (1-\beta)h > y^*$.

Theorem 4.4. *If $h > \frac{1}{a_{12}} > (1-\beta)h > y^*, a_{12} > 1, a_{21} > 1, p = 0$, then system (2.5) has an unique stable order-1 periodic solution for $\beta_* < \beta < \beta^*$ where $\beta_* = 1 - \frac{1}{a_{12}h}$.*

Proof. If $\beta_* < \beta < \beta^*$, see Fig. 10, the trajectories starting from the region G_1 will enter the region G_2 after several impulsive effects. Hence, we only consider the trajectories starting from the region G_2.

On the image set $N = \{(x, y)|y = (1-\beta)h\}$, choose an arbitrary point A_1 close to the point $(0, (1-\beta)h)$ sufficiently. The trajectory $\pi^+(A_1)$ will hit the impulsive set M at the point A_2, and then jumps to the point A_3. Since $\beta > \beta_*$, x_{A_1} small enough and $x_{A_2} = x_{A_3}$, then $x_{A_1} < x_{A_2} = x_{A_3}$. Further, we have that the successor function $f(A_1) = x_{A_3} - x_{A_1} > 0$ (see Fig. 10).

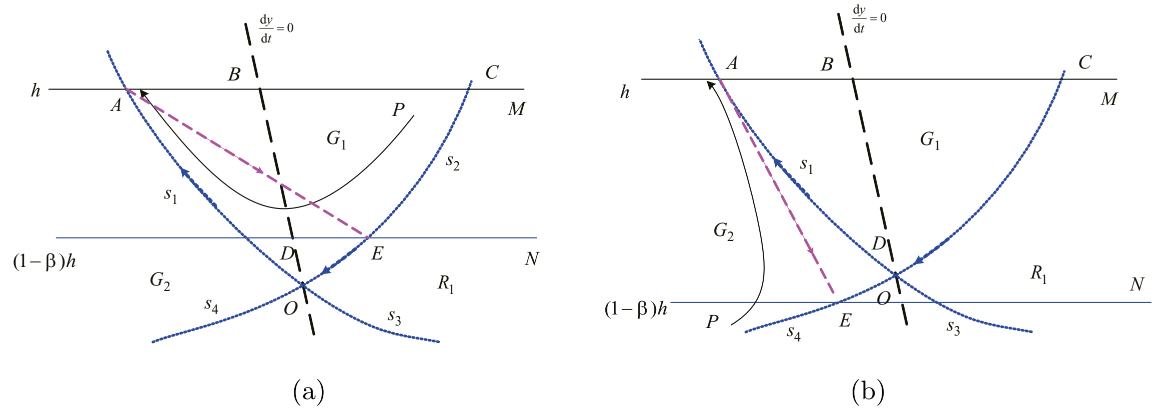

Fig. 11. Two kinds of order-1 singular cycles.

Choose a point B_1 ($B_1 \in N$) close to the separatrix s_1 sufficiently, the trajectory $\pi^+(B_1)$ hits the impulsive set M at the point B_2 and then jumps to the point B_3 which is the successor point of B_1. Since $\beta < \beta^*$ and $\frac{dx}{dt}|_{s_1} < 0$, then $x_{B_1} > x_{B_3}$. Further, the successor function $f(B_1) = x_{B_3} - x_{B_1} < 0$. Therefore, there must be a point C_1 between A_1 and B_1 such that the successor function $f(C_1) = 0$, and further system (2.5) has an order-1 periodic solution for $\beta_* < \beta < \beta^*$.

From the above discussions, we know that $|A_1B_1| > |A_3B_3|$. Since the trajectories cannot intersect, then we have

$$|A_1B_1| > |A_3B_3| > |A_5B_5| > \cdots > |A_{2k-1}B_{2k-1}| > \cdots, k = 1, 2, \ldots,$$

where A_{2k+1} and B_{2k+1} are the corresponding successor points of A_{2k-1} and B_{2k-1}, $k = 1, 2, \ldots,$ respectively. Besides, since $f(x_{A_{2k-1}}) > 0$, $f(x_{B_{2k-1}}) < 0$ and

$$\begin{aligned}|A_{2k+1}B_{2k+1}| &= x_{B_{2k+1}} - x_{A_{2k+1}} = f(x_{B_{2k-1}}) + x_{B_{2k-1}} - f(x_{A_{2k-1}}) - x_{A_{2k-1}} \\ &= f(x_{B_{2k-1}}) - f(x_{A_{2k-1}}) + |A_{2k-1}B_{2k-1}| =: F(|A_{2k-1}B_{2k-1}|),\end{aligned}$$

then $|A_{2k+1}B_{2k+1}| < |A_{2k-1}B_{2k-1}|$ and $F(|A_{2k-1}B_{2k-1}|)$ is a strictly monotonic compression map which has an unique fixed point. Further, we know that $|A_{2k-1}B_{2k-1}| \to 0$ as $k \to +\infty$ since $|A_kB_k| \geq 0$. Therefore, the order-1 periodic solution is unique if it exists.

From the bistable property and the continuous dependence of solution on the initial value, we can discuss the stability of order-1 periodic solution by discussing the successor point in the image set. Assume that the arc $\widehat{C_1C_2}$ be the trajectory of order-1 periodic solution (see Fig. 10).

For arbitrary given $\varepsilon > 0$, take $\delta = \varepsilon$ and let $\bar{A}_1 \in N$ be a point in the left δ-neighborhood of C_1, $|C_1\bar{A}_1| < \delta$. On the other hand, the point $\bar{A}_1$ must be on the line segment $\overline{A_{2k-1}B_{2k-1}}$ for some k, the successor point $\bar{A}_3$ of $\bar{A}_1$ must be on the line segment $\overline{A_{2k+1}B_{2k+1}}$. Further, as $t \to \infty$, the trajectory starting from the point $\bar{A}_1$ is close to the order-1 periodic solution C_1C_2, which implies that C_1C_2 is stable from the left side. Similarly, for the arbitrary point $\bar{B}_1$ in the right δ-neighborhood of the point C_1, its successor point $\bar{B}_3$ must lie on the line segment $C_1\bar{B}_1$. Similarly, we can know that C_1C_2 is stable from the right side. Therefore, the order-1 periodic solution C_1C_2 is stable. This completes the proof. □

5. Order-1 periodic solution for $0 < \beta < 1$ and $p > 0$

In this section, we will mainly discuss the existence of order-1 periodic solution for $0 < \beta < 1$ and $p > 0$. We first consider the existence of order-1 singular cycle.

Denote the intersection point of the image set N and the separatrixes s_2 (or s_4) by E. For a certain value of β, it is obvious that there is a value $p^* = x_E - x_A$ such that the order-1 singular cycle exists and consists of the arc $\widehat{OA}$, the line segment $\overline{AE}$, the arc $\widehat{EO}$ and the equilibrium O(see Fig. 11). System (2.5) has two kinds of order-1 singular cycles.

(1) If $(1-\beta)h > y^*$, the point near the equilibrium O moves along the separatrix s_1, reaches the point A, jumps to the point E under the impulsive effect, and then tends to the equilibrium O along the separatrix s_2 (see Fig. 11(a)).
(2) If $(1-\beta^*)h < (1-\beta)h < y^*$, the point near the equilibrium O moves along the separatrix s_1, reaches the point A, jumps to the point E under the impulsive effect, and then tends to the equilibrium O along with the separatrix s_4(see Fig. 11(b)). Here β^* is the value of β at which the singular cycle exists and $x_A = x_E$ for $p = 0$ (see Theorem 4.1).

Similarly, for a certain value $p < p^*$, there exists a corresponding value of β such that system (2.5) has an order-1 singular cycle.

When $0 < \beta < \beta^*$ and $0 < p < p^*$, the trajectories of system (2.5) starting from the region $G = G_1 + G_2$ will retain there. The followings will consider the existence of order-1 periodic solution for $0 < \beta < \beta^*$, $0 < p < p^*$.

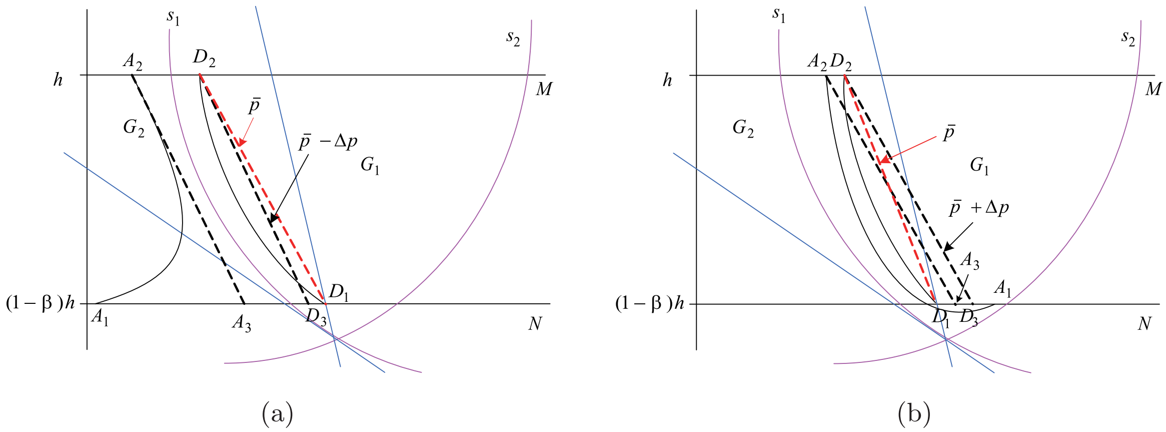

Fig. 12. The existence of two kinds of order-1 periodic solutions.

Theorem 5.1. *If $h > y^*$, $(1-\beta)h > y^*$, $a_{12} > 1$, $a_{21} > 1$, then system (2.5) has two kinds of order-1 periodic solutions for $\beta < \beta^*$ and $p < p^*$ where β^* and p^* can be seen in the above.*

Proof. Denote the intersection point of the isoclinic line $\frac{dy}{dt} = 0$ and the line $y = (1-\beta)h$ by D_1. The trajectory $\pi^+(D_1)$ hits the impulsive set M at the point D_2. Obviously, if $(1-\beta)h > y^*$, let $\bar{p} = x_{D_1} - x_{D_2}$,then the arc $\widehat{D_1D_2}$ is an order-1 periodic solution for $p = \bar{p}$.

If $p < \bar{p}$, without loss of generality, let $p = \bar{p} - \Delta p$, $0 < \Delta p < \bar{p}$, see Fig. 12(a). The successor point of D_1 is D_3, obviously, $x_{D_3} < x_{D_1}$ and the successor function $f(D_1) < 0$. Since $p > 0$, then we can easily choose a point A_1 in the image set N and close to the point $(0, (1-\beta)h)$ sufficiently, the successor point of A_1 is A_3 and $x_{A_3} > x_{A_1}$. Further, $f(A_1) > 0$. Therefore, there exists a point C between A_1 and D_1 such that $f(C) = 0$, which implies that the order-1 periodic solution exists. Similar to the proof of Theorem 4.4, we have that $|A_1D_1| > |A_3D_3| > \cdots > |A_{2k-1}B_{2k-1}| > \cdots$ and $\{|A_kD_k|\}$, $k = 1, 2, \ldots$ is a strictly monotone decreasing sequence, where A_{2k+1} and D_{2k+1} are the successor points of A_{2k-1} and D_{2k-1}, $k = 1, 2, \ldots$, respectively, then we know that the order-1 periodic solution is unique.

In the case of $p < \bar{p}$, since Δp is arbitrary, then we can know that the trajectory of order-1 periodic solution moves from right to left with the parameter p decreasing and the order-1 periodic solution is of type I.

If $p > \bar{p}$, without loss of generality, let $p = \bar{p} + \Delta p < p^*$, see Fig. 12(b). Still denote the successor point of D_1 by D_3. Obviously, $x_{D_3} > x_{D_1}$ and the successor function $f(D_1) > 0$ for $p > \bar{p}$. Since $p < p^*$, then we can choose a point A_1 be in the image set N and close to separatrix s_2 sufficiently, the successor point of A_1 is A_3 and $x_{A_3} < x_{A_1}$. Further, $f(A_1) < 0$. Therefore, there also exists a point C between A_1 and D_1 such that $f(C) = 0$ which implies that the order-1 periodic solution exists. Similar to the proof of Theorem 3.2, we know that the order-1 periodic solution is also unique.

With the value of parameter p increasing, the order-1 periodic solution tends to the separatrixes s_1 and s_2. Since the value of y first decreases and then increases, then we can know that the order-1 periodic solution is of type II.

Therefore, system (2.5) has two kinds of order-1 periodic solution for $(1-\beta)h > y^*$.This completes the proof. □

Remark 5.2. For the order-1 periodic solution of type II ($p > \bar{p}$), there exists a corresponding p' ($p' < \bar{p}$) such that the trajectory of periodic solution of type I lies on the trajectory of the periodic solution of type II (see Fig. 13).

Remark 5.3. If $(1-\beta^*)h < (1-\beta)h < y^*$, there must exist a value p^{**} of the parameter p such that all the trajectories will go into the region R after several impulsive effects. For $p < p^{**}$, the existence of order-1 periodic solution can be proved by the similar ideas of the case $(1-\beta)h > y^*$. Here omits it. It should be pointed out that all the order-1 periodic solution are of type I, and no periodic solution of type II exists.

Remark 5.4. Theorem 5.1 only states that system (2.5) has two kinds of order-1 periodic solutions. In fact, from the proof of Theorem 5.1, we can know that for the given parameters, the corresponding order-1 periodic solution is unique if it exists.

Theorem 5.5. *Suppose that system (2.5) has an order-1 periodic solution $(\tilde{x}(t), \tilde{y}(t))$ starting from the point $(x_1, (1-\beta)h)$ and hits the impulsive set M at the point (x_2, h) where $x_1 = x_2 + p$ for $p > 0$ and $0 < \beta < 1$. The order-1 periodic solution $(\tilde{x}(t), \tilde{y}(t))$ is orbitally asymptotically stable and has the asymptotic phase properties if*

$$|\mu_2| = \left|\frac{1-(1-\beta)h - a_{21}x_1}{1-h-a_{21}(x_1-p)}\,\frac{x_1-p}{x_1}\right| \exp\left(\int_0^T (-\tilde{x}(t) - b\tilde{y}(t))dt\right) < 1. \tag{5.1}$$

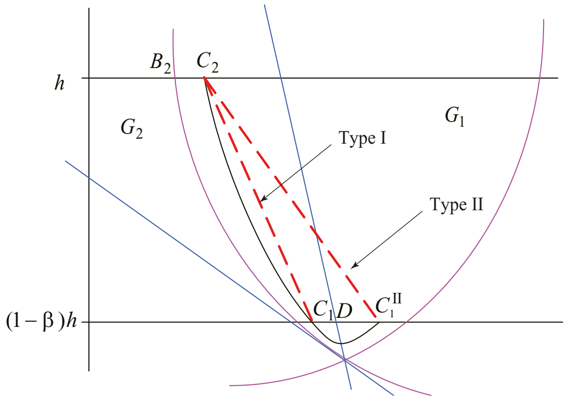

Fig. 13. The relations between the order-1 periodic solutions of types I and II.

In particular, the order-1 periodic solution of type I is orbitally asymptotically stable if one of the following conditions holds:
(1) $\beta h \le a_{21}p$;
(2) $\beta h > a_{21}p$, $h - y^* > a_{21}p$.

Proof. Let $\widetilde{x} = \widetilde{x}(t), \widetilde{y} = \widetilde{y}(t)$. According to Lemma 2.5, we can calculate and obtain that

$$\frac{\partial B}{\partial x} = 0, \quad \frac{\partial B}{\partial y} = -\beta, \quad \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial \phi}{\partial y} = 1, \quad \frac{\partial A}{\partial x} = 0, \quad \frac{\partial A}{\partial y} = 0,$$

$$\Delta_1 = \frac{Q^+}{Q} = \frac{b(1-\beta)h(1-(1-\beta)h - a_{21}x_1)}{bh(1-h-a_{21}x_2)} = \frac{(1-\beta)(1-(1-\beta)h - a_{21}x_1)}{1-h-a_{21}(x_1-p)},$$

and

$$\frac{\partial P}{\partial x}(\tilde{x}(t), \tilde{y}(t)) + \frac{\partial Q}{\partial y}(\tilde{x}(t), \tilde{y}(t)) = 1 - 2\tilde{x} - a_{12}\tilde{y} + b - 2b\tilde{y} - a_{21}b\tilde{x}$$
$$= \frac{d\tilde{x}}{\tilde{x}dt} - \tilde{x} + \frac{d\tilde{y}}{\tilde{y}dt} - b\tilde{y} = \frac{d\tilde{x}}{\tilde{x}dt} + \frac{d\tilde{y}}{\tilde{y}dt} - \tilde{x} - b\tilde{y},$$

then

$$\exp\left[\int_0^T \left(\frac{d\tilde{x}}{\tilde{x}dt} + \frac{d\tilde{y}}{\tilde{y}dt} - b\tilde{y}\right)dt\right] = \exp\left(\int_{x_1}^{x_1-p} \frac{d\tilde{x}}{\tilde{x}}\right)\exp\left(\int_{(1-\beta)h}^{h} \frac{d\tilde{y}}{\tilde{y}}\right)\exp\left[\int_0^T (-\tilde{x} - b\tilde{y})dt\right]$$
$$= \frac{x_1-p}{x_1}\frac{1}{1-\beta}\exp\left(\int_0^T (-\tilde{x} - b\tilde{y})dt\right).$$

Further,

$$\mu_2 = \frac{(1-\beta)(1-(1-\beta)h - a_{21}x_1)}{1-h-a_{21}(x_1-p)}\frac{x_1-p}{x_1}\frac{1}{1-\beta}\exp\left(\int_0^T (-\tilde{x} - b\tilde{y})dt\right)$$
$$= \frac{1-(1-\beta)h - a_{21}x_1}{1-h-a_{21}(x_1-p)}\frac{x_1-p}{x_1}\exp\left(\int_0^T (-\tilde{x} - b\tilde{y})dt\right).$$

According to Lemma 2.5, the order-1 periodic solution is orbitally asymptotically stable and has the asymptotic phase properties if (5.1) holds.

Let

$$g(x_1) = \frac{1-(1-\beta)h - a_{21}x_1}{1-h-a_{21}(x_1-p)} = \frac{1-h-a_{21}x_1+\beta h}{1-h-a_{21}x_1+a_{21}p} = 1 + \frac{\beta h - a_{21}p}{1-h-a_{21}x_1+a_{21}p}.$$

For the order-1 periodic solution of type I, since $\frac{dy}{dt}|_{(x_1,(1-\beta)h)} = 1-h-a_{21}x_1 > 0$, we know that $g(x_1) \le 1$ if $\beta h \le a_{21}p$. Further, $0 < \mu_2 < 1$ if $\beta h \le a_{21}p$ and the order-1 periodic solution is orbitally asymptotically stable.

If $\beta h > a_{21}p$, from $\frac{\partial g(x_1)}{\partial x_1} = \frac{a_{21}(\beta h - a_{21}p)}{(1-h-a_{21}x_1+a_{21}p)^2} > 0$, for the periodic solution of type I, since $x_1 < x^*$ and

$$g(x_1) < 1 + \frac{\beta h - a_{21}p}{1-h-a_{21}x^*+a_{21}p} = 1 + \frac{\beta h - a_{21}p}{y^*-h+a_{21}p} = 1 - \frac{\beta h - a_{21}p}{h-y^*-a_{21}p}.$$

Then $g(x_1) < 1$ for $\beta h > a_{21}p$ and $h - y^* > a_{21}p$. Further, $0 < \mu_2 < 1$ and the order-1 periodic solution is orbitally asymptotically stable. This completes the proof. □

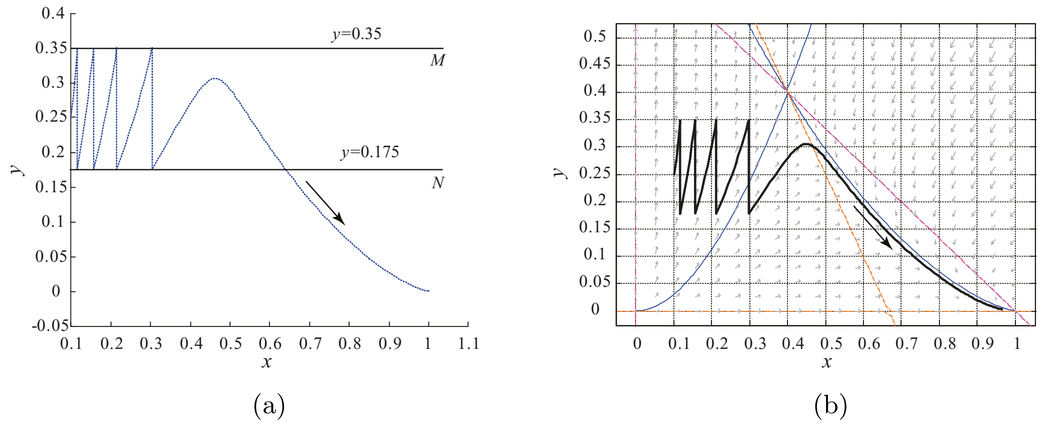

Fig. 14. The phase portrait of system (2.5) for $h = 0.35 < y^* = 0.4, \beta = 0.5$.

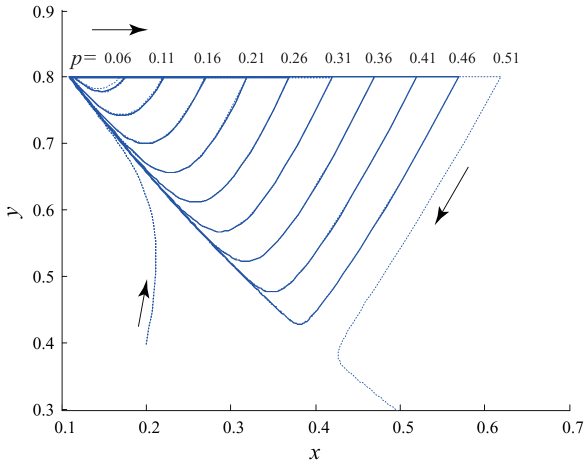

Fig. 15. The phase portraits of system (2.5) for $h = 0.8, \beta = 0$ and $p = 0.06 : 0.05 : 0.51$, respectively.

6. Numerical simulations and conclusions

To verify the mathematical results obtained above, we take $a_{12} = a_{21} = 1.5 > 1$ and $b = 2$, then system (2.5) is a bistable system and has two stable equilibria (1,0) and (0,1), a positive equilibrium $(x^*, y^*) = (0.4, 0.4)$ which is a saddle point.

Theorem 2.6 shows that the solutions of system (2.5) tends to the equilibrium (1,0) after at most finite times impulsive effects. We take $h = 0.35 < y^* = 0.4, \beta = 0.5, p = 0$, then the phase portrait of the solution starting from the point (0.1, 0.25) can be seen in Fig. 14(a). Fig. 14(b) is given to clearly see the position of the trajectory in the phase plane. From Fig. 14, we can see that the trajectory tends to (1,0) after four times impulsive effects. For the case of $p \neq 0$, the similar result can be found and the phase portrait is omitted here. This implies that the threshold h should be larger than y^* to reach the aim of control, that is, to maintain the biomass of two species in a suitable region. Otherwise, the control is not effective.

In the following, we gives the numerical simulations of the case $h > y^*$. Theorem 3.2 shows that there exists a value p_2^* of the parameter p such that system (2.5) has an order-1 periodic solution for $\beta = 0$ and $p < p_2^*$. In order to verify the existence of p_2^*, we take $h = 0.8 > y^* = 0.4, \beta = 0$ and let $p = 0.06 : 0.05 : 0.51$(that is, the value of p is taken from 0.01 to 0.51 by step size 0.05, respectively), then the phase portraits of the solutions starting from the same initial value $(0.2, 0.4) \in G_2$ can be seen in Fig. 15. From Fig. 15, we can see that system (2.5) has an order-1 periodic solution for $p \leq 0.46$, but the solution tends to (1,0) for $p \geq 0.51$, which implies that there must be a critical value $p_2^*(0.46 < p_2^* < 0.51)$ such that system (2.5) has an order-1 periodic solution for $p < p_2^*$, and the trajectories tend to (1,0) for $p > p_2^*$. Therefore, for the control strategy of single releasing fish, if we want to maintain the biomass of two species in the region G_1, the amount p of the released fish should be chosen suitably and not be larger than p_2^*. Otherwise, it can result in the extinction of the algae species y. Besides, for $p < p_2^*$, we can also see that the periodic orbits tend to the separatrixes s_1 and s_2 with p increasing. Fig. 15 also shows that the biomass of the algae first decreases and then increases after impulsive effect, and the periodic solutions are of type II.

Theorem 4.2 shows that the boundary order-1 periodic solution $(0, \tilde{y}(t))$ of system (2.5) is stable if $(1-\beta)h \geq \frac{1}{a_{12}}$, unstable if $(1-\beta)h < \frac{1}{a_{12}}$. We take $h = 0.8 > y^*, p = 0$, Fig. 16(a) shows the solution tends to the boundary periodic solution $(0, \tilde{y}(t))$

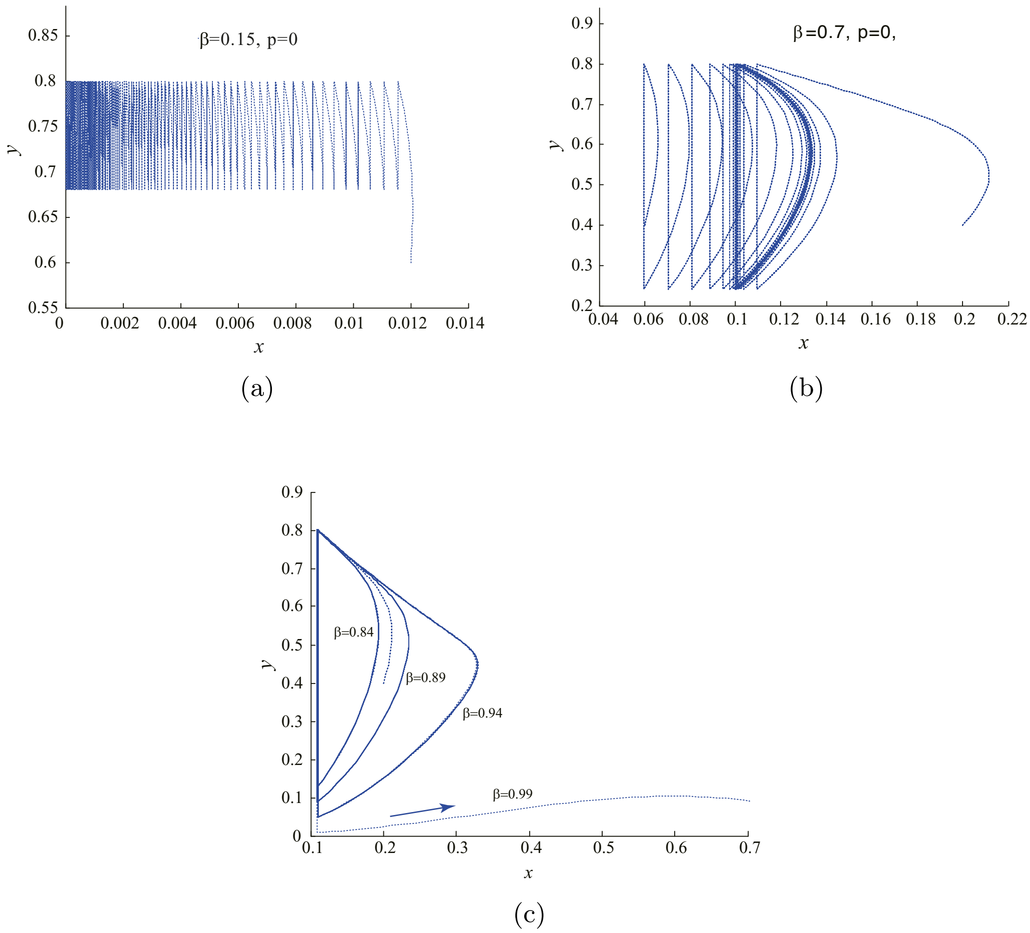

Fig. 16. The phase portraits of system (2.5) for $h = 0.8,\ p = 0$.

where $\beta = 0.15,\ (1-\beta)h = 0.68 > 1/a_{12} \approx 0.67$. Fig. 16(b) shows that there exists a positive order-1 periodic solution for $\beta = 0.7$ where $(1-\beta)h = 0.24 < 1/a_{12}$. Both trajectories starting from (0.06,0.4) and (0.2,0.4) tend to the order-1 periodic solution from the left side and the right side, respectively. Theorem 4.1 implies that there is a value β^* of the parameter β such that the solutions of system (2.5) tend to (1,0) for $\beta > \beta^*$. So we take $\beta = 0.84 : 0.05 : 0.99$, then Fig. 16(c) shows that system (2.5) has order-1 periodic solutions for $\beta \le 0.94$, and tends to (1,0) for $\beta = 0.99$. This implies that there must be a value $\beta^* \in (0.94, 0.99)$ such that system (2.5) has an order-1 periodic solution for $\beta < \beta^*$ and the trajectory tends to (1,0) after one impulsive effect for $\beta > \beta^*$.

From Fig. 16, we can see that if $h > 1/a_{12} > y^*$, then there are two thresholds $\beta_* = 1/a_{12}$ and $\beta^* \in (0.94, 0.99)$ such that system (2.5) has three cases: system (2.5) has a boundary periodic solution for $\beta < \beta_*$, a positive periodic solution for $\beta_* < \beta < \beta_*$ and the solutions tend to (1,0) for $\beta > \beta^*$. This implies that for the single chemical control, if the harvest rate of the algae is smaller than β_*, then the control is also not effective and the fish still tends to be extinct. When $\beta > \beta^*$, the algae species y tends to be extinct. Only the harvest rate is lager than β_* and less than β^*, then the aim of control is reached. Therefore, for the single chemical control of spraying algaecide, the harvest rate should be chosen carefully. From Theorem 4.4, we know that there exists a positive order-1 periodic solution for $p = 0$ and $\beta_* < \beta < \beta^*$. Although so, the biomass of species x is smaller, which may be a part of the reason why the algae always increases after spraying algaecide.

If $p > 0$ and $0 < \beta < 1$, Fig. 17 gives the phase portraits for $h = 0.8,\ \beta = 0.15,\ p = 0.01 : 0.05 : 0.46$, respectively, where $(1-\beta)h > y^*$, and shows that system (2.5) has the positive order-1 periodic solutions, different values of p correspond to different positions of the trajectories for $p \le 0.41$. But the solutions tend to (1,0) if $p \ge 0.46$. Thus, there is a critical value $p^* \in (0.41, 0.46)$ such that the solutions will tend to (1, 0) for $p > p^*$. When $p < p^*$, the order-1 periodic solution exists. The parameters in Fig. 17 are the same as that in Fig. 16(a) except p. Fig. 16(a) shows that the solution tends to the boundary solution $(0, \widetilde{y}(t))$ and the species x tends to be extinct for $\beta < \beta^*$. But in this case, if the releasing measure is also taken, then the positive periodic solution exists. This implies that the combination of two control measures is more effective than the single chemical control and the critical amount of the released fish is also decreased.

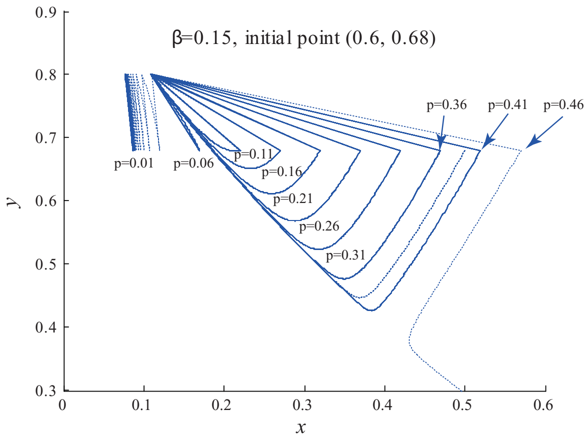

Fig. 17. The phase portraits of system (2.5) for $h = 0.8$, $\beta = 0.15$, $p = 0.01 : 0.05 : 0.46$, respectively.

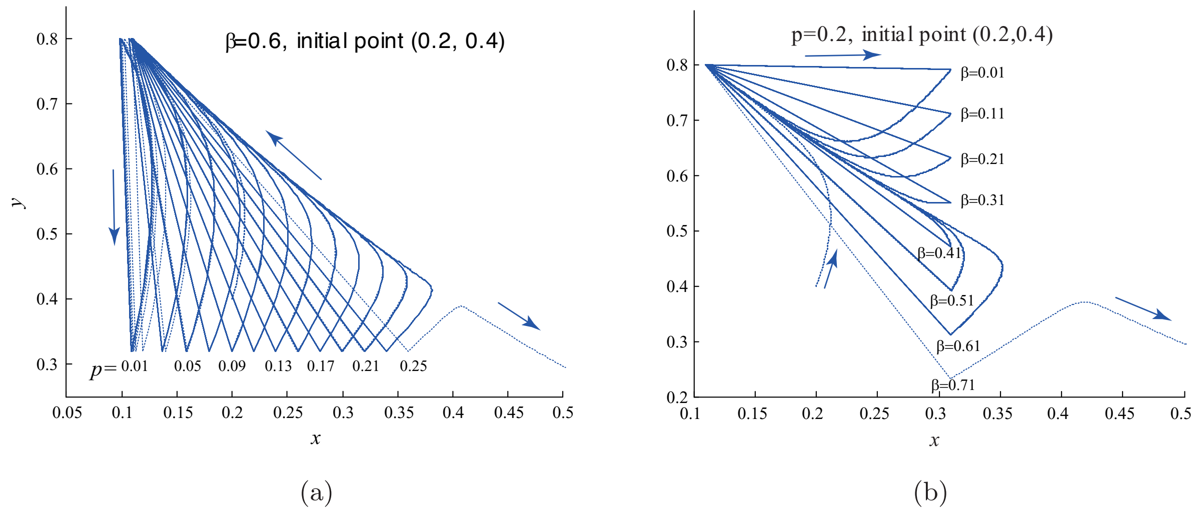

Fig. 18. The phase portraits of system (2.5) for $\beta = 0.15$ and $p = 0.01 : 0.01 : 0.11$, $p = 0.2$ and $\beta = 0.01 : 0.1 : 0.71$, respectively.

Fig. 18(a) gives the numerical simulations for $h = 0.8$, $\beta = 0.6$ and $p = 0.01 : 0.02 : 0.25$, respectively, where $(1 - \beta)h < y^*$. Fig. 18(b) shows the numerical simulations for $h = 0.8$, $p = 0.2$ and $\beta = 0.01 : 0.1 : 0.71$, respectively. From Figs. 17 and 18, we know that for a given parameter β (or p), there exists a corresponding value p^*(or β^*) such that system (2.5) has the order-1 periodic solution for $p < p^*$(or $\beta < \beta^*$), the solutions tend to (1,0) for $p > p^*$(or $\beta > \beta^*$).

From those mathematical and numerical results, we know that, for a system with bistable property, the control threshold h should be given suitably. In addition, different control parameters can result in different results. For the control parameters p and β, there must be the corresponding thresholds p^* and β^* such that system (2.5) has order-1 periodic solution if $p < p^*$ and $\beta < \beta^*$, the solutions tend to (1,0) for $p > p^*$ and $\beta > \beta^*$. Since system (2.5) does not have the explicit expression of solution, then we only give the existence of the thresholds p^* and β^*, and cannot give the explicit expressions of them. Since the curves, including s_1, s_4 and the equilibrium O, is monotone, then we can know that for the parameter $\beta_* < \beta < \beta^*$, if β increases, then the threshold p^* decreases correspondingly.

Those mathematical results show that it is difficult for people to control the algae bloom in an eutrophic water body because there are the critical values for the control parameters h, p and β. Therefore, for the bistable system, the control parameters should be chosen carefully. If the suitable parameters are given, then the positive periodic solution exists and two species coexist.

If we pay our attentions to the algae and its tendency of evolution after the impulsive control is performed, then we can see that, for the single spraying algaecide ($p = 0$), the amount of the algae is always increasing in the period of control. For the single releasing fish ($\beta = 0$), the amount of the algae first decreases and then increases. The integrated control combining spraying algaecide and releasing fish has two cases above. For both the single releasing fish and the integrated control, the amount of the algae decreases after the control measures are performed. Since the chemical control can quickly decrease the amount of the algae, then we think that the integrated control should be the preferred strategy (if the algaecide has less harmful to other aquatic organisms).

From the above discussions, we know that the critical values of the combination control are less than the single control measures. Whether there is the optimal harvest rate β and the amount p of the released fish such that the control effect is best and the cost of control is lowest. In addition, how to give the objective function? There are some troubles for us to discuss the above optimal problems which will be our future work.

Acknowledgment

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