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Qualitative analysis of a korean pine forest model with impulsive thinning measure ☆

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ABSTRACT

A korean pine forest model with impulsive thinning measure is presented by using impulsive state feedback system to investigate the periodicity of the regeneration process of the forest. Based on the qualitative properties of the corresponding continuous system, the existences of order-1 periodic solutions are discussed. If the positive equilibrium of the continuous system is globally stable, then the impulsive state feedback system has an order-1 periodic solution and no order-$k(k \geqslant 2)$ periodic solution. The conditions for the orbitally asymptotical stability of order-1 periodic solution are given and discussed by the analogue of the Poincaré criterion. For the case that the continuous system has a stable limit cycle, the existence of order-1 periodic solution of the impulsive state feedback system are also discussed, the results show that there are three kinds of order-1 periodic solutions. Finally, the mathematical results are verified by the numerical simulations. Moreover, the numerical results show that the impulsive state feedback system has order $k(k \geqslant 1)$ periodic solutions in the interior of the limit cycle of the continuous system for some parameters.

1. Introduction

Korean pine (*Pinus koraiensis Sieb.et Zucc.*) is a precious and rare tree species and mainly distributes in Changbai Mountain and Xingan Mountain areas of China. A few of them distribute in some areas of Japan, Korea and Russia. To protect the korean pine forest and maintain its regeneration and succession, the management measures such as quantitative thinning, thinning and single tree selective cutting are taken. In those measures, the quantitative thinning is a main measure because it can not only protect the species but also make the managers obtain economic benefits.

Some references have studied the storage of fallen trees of korean pine mixed forest [1], population structure and regeneration mode [2]. A few of references have studied the dynamical behaviors of korean pine forest models. For examples, Ref. [3] developed succession and silviculture model for broad-leaved pinus koriensis in Changbai Mountain by combining the framework of ZELIG and characteristics of broad-leaved pinus koriensis forests in Changbai area. Ref. [4] studied the structure of food net, habitat conditions, nature regeneration, the species structure of young forest and mature forest, and

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gave the regeneration model of korean pine. Ref. [5] have studied the wave features of population changes of korean pine in natural forest. To consider the dynamical properties of the regeneration model, Ref. [6] presented and studied a kind of mathematical model of population age replace of korean pine in natural forest. In paper [6], the conditions for the existences and stabilities of the equilibrium and the limit cycle are given.

Considering the current level of korean pine, the strategies of tending and thinning was taken in Changbai Mountain and Xingan Mountain areas. The main purpose of tending and thinning is to implement the forest, adjust the stand density, improve stand condition, improving forest quality, enhancement and play a variety of beneficial function of forest. Besides, through the thinning activities, the production of a certain number of wood is to meet the needs of national construction and people's life. But the time and the total yield of thinning depend on the state of the forest. Therefore, a model of korean pine with impulsive thinning is presented to describe the regeneration process of the forest under mankind's management measure.

There are some papers investigating the biological and mathematical model with impulsive control. For example, Ref. [7,8] discussed the pest models with impulsive control. Ref. [9,10] discussed the periodic solution of two microorganism culture systems with impulsive state feedback control by the existence criteria of periodic solution of a general planar impulsive autonomous system. Ref. [11] considered the system with impulsive state feedback control as semi-continuous dynamical system, and gave the definitions and some methods to discuss the qualitative problems of the models. As the applications of semi-continuous dynamical system, papers [12–15] etc. gave the preliminary results about the biomathematical model with impulsive state feedback control. In this paper, we will discuss a kind of korean pine model with impulsive thinning measure, which depends on the state of the species population, and show the periodicity of the regeneration of the forest.

2. Model formulation and preliminary

Paper [6] presented the following model to study the population age replace of korean pine forest.

$$\begin{cases} \frac{dx}{dt} = x\left(1 - \frac{x}{k}\right) - \frac{axy}{x+c}, \\ \frac{dy}{dt} = bxy - dy, \end{cases} \tag{2.1}$$

where x and y denote the population of sapling and seed trees (that is, young trees and mature trees),respectively. Parameters k, a, b, c and d are positive. System (2.1) assumed that

(H1) The intrinsic rate of increase of the sapling trees satisfies the Logistic function $\left(\text{i.e. } x\left(1 - \frac{x}{k}\right)\right)$, the restriction from the seed trees is similar to the Holling type II functional response, that is, $-\frac{x}{x+c}$.
(H2) The growth rate of seed trees is proportion to the sapling, that is to say, more sapling trees and more seed trees.

Since the population of seed trees reaches some levels, it will decrease the growth rate of saplings trees, then the harvest of seed trees is necessary to maintain the growth of sapling trees. At the same time, the moderate thinning of the seed trees can increase the number of the big diameter wood and the production of dimension lumbers, and then increases the benefits of the managers. The thinning measures of seed trees have constant thinning and proportional thinning. In this paper, we only consider the thinning measure of the proportional harvest, that is, when the population of the seed trees reaches a threshold level (denoted by h), the seed trees is harvested by proportional thinning measure, and the harvest rate is denoted by $p(0 < p < 1)$. Incorporating the thinning measure, system (2.1) becomes the following forms:

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= x\left(1 - \frac{x}{k}\right) - \frac{axy}{x+c}, \\ \frac{dy}{dt} &= bxy - dy, \end{aligned}\right\} & y < h, \\ \left.\begin{aligned} \Delta x &= 0, \\ \Delta y &= -py, \end{aligned}\right\} & y = h, \end{cases} \tag{2.2}$$

where $\Delta x = x^+ - x, \Delta y = y^+ - y$. Denote the impulse set of system (2.2) by $M = \{(x, y) | y = h\}$ and the image set of the impulse set M by $N = \{(x, y) | y = (1 - p)h\}$. Throughout the paper, we assume that $x(0) > 0$ and $0 < y(0) < h$.

In the following, we will discuss the order-$k(k \geqslant 1)$ periodic solution of system (2.2) to show the periodicity and stability of the regeneration korean pine of the forest under the impulsive thinning measure. Next, we give the basic properties of system (2.1) which is the continuous subsystem of system (2.2).

Lemma 2.1 [6]. *If $bk > d$, then system (2.1) has three equilibria:* $(0, 0), (k, 0)$ *and* (x^*, y^*), *where*

$$x^* = \frac{d}{b}, \quad y^* = \frac{1}{a}\left(c + \frac{d}{b}\right)\left(1 - \frac{d}{bk}\right).$$

The equilibria $(0, 0)$ *and* $(k, 0)$ *are saddle points. The point* (x^*, y^*) *is unstable if* $bk - bc - 2d > 0$, *stable if* $bk - bc - 2d \leqslant 0$.

Lemma 2.2 [6]. *If $bk - bc - 2d \leqslant 0$, then equilibrium (x^*, y^*) is globally stable.*

Lemma 2.3 [6]. *If $kb - bc - 2d < 0$, then system (2.1) has no limit cycle and closed orbit in $R^+ = \{(x, y)|x > 0,\ y > 0\}$.*

Lemma 2.4 [6]. *System (2.1) has a unique stable limit cycle in $R^+ = \{(x, y)|x > 0,\ y > 0\}$ if and only if $kb - bc - 2d > 0$.*

The illustration of vector fields of system (2.1) can be seen in Fig.1, where L_1 and L_2 are the isoclines, that is, $L_1 : y = \frac{1}{a}(x + c)(1 - \frac{x}{k})$ and $L_2 : x = x^*$.

System (2.2) is of impulsive semi-dynamical system. The definitions and results of impulsive semi-dynamical system can be found in Ref. [16]. Here only gives the definition of order-$k(k \geqslant 1)$ periodic solution. More details can be seen in Ref. [16] and Ref. [11].

Definition 2.5. (Lakshmikantham, et al. [16]). A trajectory $\widetilde{\pi}_x$ is said to be periodic of order k if there exist positive integers $m \geqslant 1$ and $k \geqslant 1$ such that k is the smallest integer for which $x^+_m = x^+_{m+k}$.

In order to investigate the qualitative properties of system (2.2), we need introduce the successor function. For system (2.2), we define the successor function as follows:

Definition 2.6. Let M and N be the lines where the impulse set and its image set lies on respectively (see Fig.2). We define a new coordinate axis O' on the line N, the direction and length unit of the new coordinate axis are the same as that of the axis-x. For any point $A(x, y) \in N,\ x > 0,\ y > 0$, the new coordinate of $A(x, y)$ denotes by $l(A)$ and $l(A) = x$.

For any point $A(x_0, y_0) \in N$, the trajectory of system (2.2) starting from the point A hits the impulse set M, and then jumps to $A_1(x_1, y_1) \in N, y_0 = y_1 = (1 - p)h$, then the point A_1 is said to the success point of A, and the success function can be written as $f(A) = l(A_1) - l(A_0) = x_1 - x_0$.

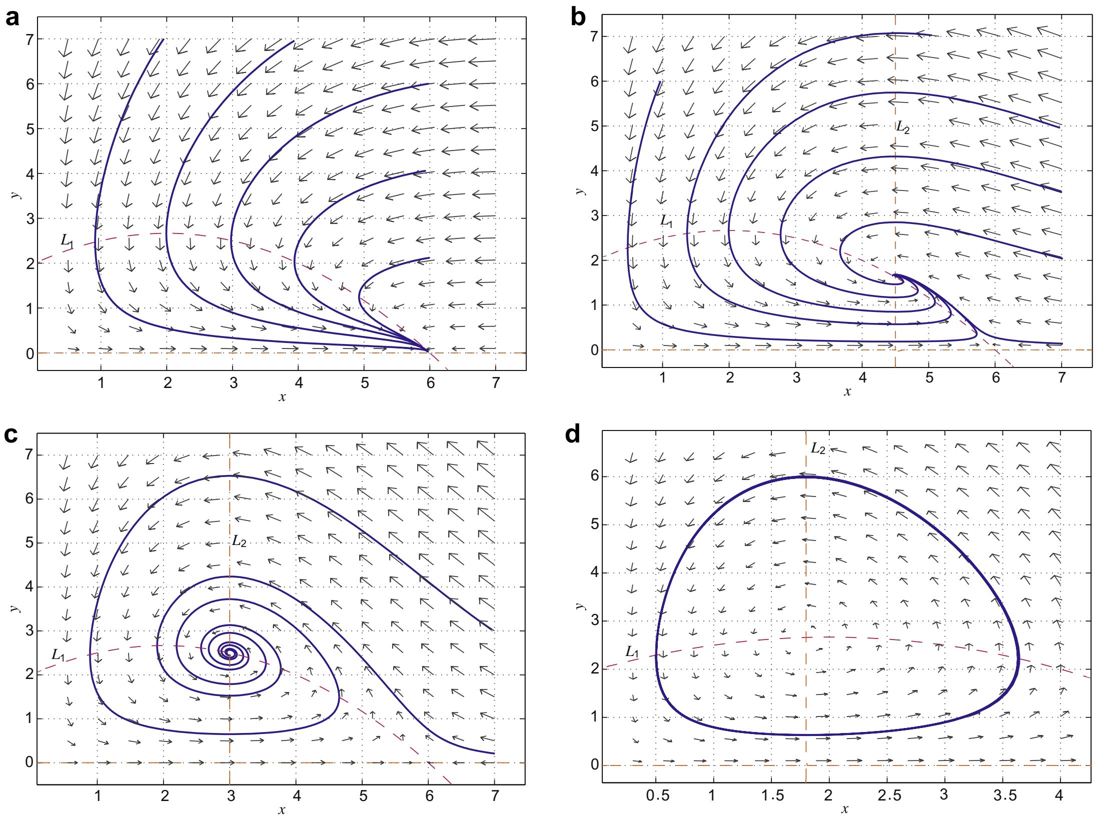

Fig. 1. The illustration of vector fields of system (2.1). a. Equilibrium $(k, 0)$ is stable; b. Equilibrium (x^*, y^*) is a stable node; c. Equilibrium (x^*, y^*) is a stable focus; d. System (2.1) has an limit cycle.

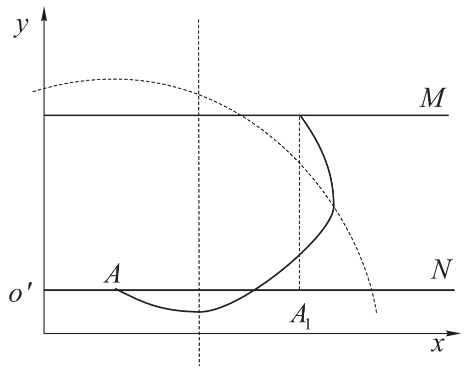

Fig. 2. The illustration of the success function of system (2.1).

Lemma 2.7 (*Analogue of the Poincaré criterion [17]*). *The T-periodic solution $x = \xi(t),\ y = \eta(t)$ of the system*

$$\begin{cases} \frac{dx}{dt} = P(x,y), \frac{dy}{dt} = Q(x,y), & \text{if} \quad \phi(x,y) \neq 0, \\ \Delta x = A(x,y), \Delta = B(x,y), & \text{if} \quad \phi(x,y) = 0 \end{cases}$$

is orbitally asymptotically stable and enjoys the property of asymptotic phase if the multiplier μ_2 satisfies the condition $|\mu_2| < 1$, where

$$\mu_2 = \prod_{k=1}^{q} \Delta_k \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\right) dt\right],$$

$$\Delta_k = \frac{P_+\left(\frac{\partial B}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial B}{\partial x}\frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial x}\right) + Q_+\left(\frac{\partial A}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right)}{P\frac{\partial \phi}{\partial x} + Q\frac{\partial \phi}{\partial y}}$$

and $P, Q, \frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial B}{\partial x}, \frac{\partial B}{\partial y}, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}$ are calculated at the point $(\xi(\tau_k),\ \eta(\tau_k))$ and $P_+ = P(\xi(\tau_k^+),\ \eta(\tau_k^+)),\ Q_+ = Q(\xi(\tau_k^+),\ \eta(\tau_k^+))$.

3. The existence of order-1 periodic solution for $bk - bc - 2d \leqslant 0$

From Lemma 2.2, we know that the positive equilibrium (x^*, y^*) is globally stable if $bk - bc - 2d \leqslant 0$. In the following, we discuss the periodic solution for the cases of $h < y^*$ and $h > y^*$, respectively.

3.1. The case of $h < y^*$

Theorem 3.1. *If $h < y^*$, them system (2.2) has a unique order-1 periodic solution.*

Proof. If $h < y^*$, then the line $y = (1-p)h$ intersects the isoclinic lines L_2 and L_1 at the point $A(x_A, (1-p)h)$ and $B(x_B, (1-p)h)$ respectively (see Fig.3). The trajectory from the point A hits the impulse set M at the point A^1 and then jumps to the point $A_1(x_{A_1}, (1-p)h) \in N = \{(x,y)|y = (1-p)h\}$, so the point A_1 is the success point of A, the success function $f(A) = l(A_1) - l(x_A)$. Clearly, $f(A) = x_{A_1} - x_A > 0$. On the other hand, the trajectory starting from the point $B(x_B, h)$ hits the impulse set M at the point B^1 and then jumps to the point $B_1(x_{B_1}, h) \in N$, so the point B_1 is the success point of B, the success function $f(B) = l(B_1) - l(x_B)$. Clearly, $f(B) = x_{B_1} - x_B < 0$. Therefore, there must exist a point $C(C \in N)$ between A and B such that $f(C) = l(x_C) - l(x_{C_1}) = 0$. That is to say, the trajectory starts from the point C is an order-1 periodic solution of system (2.2).

Since the trajectories starting from the points in the sets $\{(x,y)|0 < x < x_A,\ y = (1-p)h\}$ and $\{(x,y)|x > x_B,\ y = (1-p)h\}$ will enter the set $AB = \{(x,y)|x_A \leqslant x \leqslant x_B,\ y = (1-p)h\}$ after several times impulsive effects at most, then the initial point of the order-1 periodic solution only lies in the set $AB = \{(x,y)|x_A < x < x_B,\ y = (1-p)h\}$.

The set $AB = \{(x,y)|x_A \leqslant x \leqslant x_B,\ y = (1-p)h\}$ is mapped to the set $A^1B^1 = \{(x,y)|x_{A^1} < x < x_{B^1},\ y = h\}$ by the first and second equations of system (2.2). Subsequently, the set A^1B^1 is mapped to the set $A_1B_1 = \{(x,y)|x_{A_1} < x < x_{B_1},\ y = (1-p)h\}$ by the third and fourth equations of system (2.2). Since $h < y^*$, it is easily to know that $x_A < x_{A_1},\ x_{B_1} < x_B$, the line segments $\overline{AB}$ and $\overline{A_1B_1}$ satisfy $|\overline{AB}| > |\overline{A_1B_1}|$. We continue the above process and know from the vector fields of system (2.2) that

$$|\overline{AB}| > |\overline{A_1B_1}| > |\overline{A_2B_2}| > \cdots$$

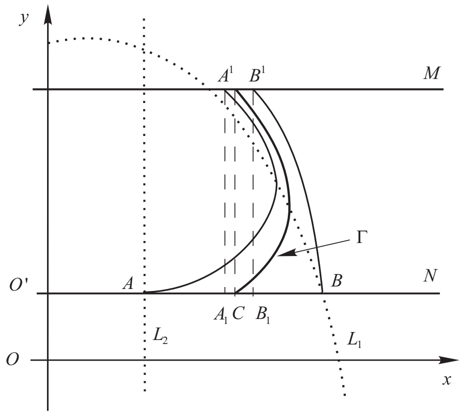

Fig. 3. The illustration of existence of order-1 periodic solution of system (2.1).

and

$$x_A < x_{A_1} < x_{A_2} < \cdots < x_{B_2} < x_{B_1} < x_B.$$

Therefore, the sequence $|\overline{A_nB_n}|$ is convergent monotonously and $\lim_{n\to\infty}|\overline{A_nB_n}| = 0$, which implies that there exists a unique point C such that $f(C) = l(x_C) - l(x_{C_1}) = 0$. Furthermore, system (2.2) has a unique order-1 periodic solution for $h < y^*$. This completes the proof. □

Theorem 3.2. *The order-1 periodic solution of system (2.2) is orbitally asymptotically stable and enjoys the property of asymptotic phase if $h < y^*$ and*

$$-x^*(k+c)^2 + ak^2h(k+c) - kac(1-p)h < 0.$$

In particular, if $h < h^* = \frac{k+c}{a^2}x^*$, the order-1 periodic solution is orbitally asymptotically stable if it exists.

Proof. According to Lemma 2.7, let Γ be the order-1 periodic solution, $(\xi, (1-p)h) \in N$ and $(\xi_1, h) \in M$. So we have

$$P(x,y) = x\left(1 - \frac{x}{k}\right) - \frac{axy}{x+c}, \quad Q(x,y) = bxy - dy,$$

$$A(x,y) = 0, B(x,y) = -py, \quad \phi(x,y) = y - h.$$

Furthermore,

$$P_+ = \xi\left(1 - \frac{\xi}{k}\right) - \frac{a\xi(1-p)h}{\xi + c}, \quad Q_+ = (b\xi - d)(1-p)h, \quad Q = (b\xi - d)h$$

and

$$\frac{\partial A}{\partial x} = 0, \quad \frac{\partial A}{\partial y} = 0, \quad \frac{\partial B}{\partial x} = 0, \quad \frac{\partial B}{\partial y} = -p, \quad \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial \phi}{\partial y} = 1,$$

then $\Delta_1 = \frac{Q_+}{Q} = 1 - p$.

Since

$$\frac{\partial P}{\partial x} = x - \frac{2x}{k} - \frac{acy}{(x+c)^2} = \frac{\dot{x}}{x} - \frac{x}{k} + \frac{axy}{x+c} - \frac{acy}{(x+c)^2},$$

$$\frac{\partial Q}{\partial y} = bx - d = \frac{\dot{y}}{y},$$

$$\int_0^T \frac{\partial Q}{\partial y} \mathrm{d}t = \int_0^T \frac{\mathrm{d}y}{\mathrm{d}t}\frac{1}{y}\mathrm{d}t = \int_{(1-p)h}^{h} \mathrm{d}(\ln y) = \ln\frac{1}{1-p},$$

and $\int_0^T \frac{\dot{x}}{x}\mathrm{d}t = \int_\xi^\xi \mathrm{d}(\ln x) = 0$, then

$$\int_0^T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right)\mathrm{d}t = \ln\frac{1}{1-p} + \int_0^T \left(-\frac{x}{k} + \frac{axy}{x+c} - \frac{acy}{(x+c)^2}\right)\mathrm{d}t,$$

and

$$\mu_2 = \exp\left(\int_0^T \left(-\frac{x}{k} + \frac{axy}{x+c} - \frac{acy}{(x+c)^2}\right)\mathrm{d}t\right).$$

For any point $(x,y) \in \Gamma$, since $x^* < x < k$ and $(1-p)h < y < h$, then

$$-\frac{x}{k} + \frac{axy}{x+c} - \frac{acy}{(x+c)^2} < -\frac{x^*}{k} + \frac{akh}{k+c} - \frac{ac(1-p)h}{(k+c)^2} < 0$$

if $-x^*(k+c)^2 + ak^2h(k+c) - kac(1-p)h < 0$. Furthermore, $0 < \mu_2 < 1$.

In particular, if $h < h^*$, then $-\frac{x^*}{k} + \frac{akh}{k+c} < 0$ and $0 < \mu_2 < 1$, furthermore the order-1 periodic solution is orbitally asymptotically stable if it exists. This completes the proof. □

3.2. The effects of the harvest rate p on order-1 periodic solution

Let C_1C^1 be the order-1 periodic solution Γ_1 for $p = p_1$, the image set $N_1 = \{(x,y)|y = h_1,\ h_1 = (1-p_1)h\}$, where $C_1 = \Gamma_1 \bigcap N_1,\ C^1 = \Gamma_1 \bigcap M$. Without loss of generality, let $p_2 > p_1$, then $N_2 = \{(x,y)|y = h_2,\ h_2 = (1-p_2)h\}$ (see Fig. 4.

When the harvest rate p is increased, then the point C^1 is mapped to the point A_1. According to system (2.2), the trajectory starting from the point A_1 goes to the point A^1 and then jumps to the point A_2 under the impulsive effect. It is easily know that A_2 is the success point of the point A_1 and the success function $f(A_1) = l(A_2) - l(A_1) > 0$. Denote the intersection point of N_2 and the isoclinic line $L_1 : \mathrm{d}x/\mathrm{d}t = 0$ by B_2. Similar to the proof of Theorem 3.1, we know that there exists a unique point C_2 between A_1 and B_2 such that $f(C_2) = 0$, that is, there is a unique periodic solution for $p = p_2$, $C_2 = \Gamma_2 \bigcap N_2,\ C^2 = \Gamma_2 \bigcap M$. Clearly, from the above discussion, we know that the point of order-1 periodic solution on the impulse set $M = \{(x,y)|y = h\}$ moves from C^1 to C^2 with the increase of the harvest rate p, that is, $x_{C^1} < x_{C^2}$.

3.3. The case of $h > y^*$

Since the point $(k,0)$ is a saddle point for $bk - kc - 2d \leqslant 0$, then there is a saddle separatrix denoted by l_0 (see Fig. 5). Furthermore,we know that l_0 intersects with the isoclinic line $L_2 : x = x^*$ since $\mathrm{d}x/\mathrm{d}t < 0$ for the point $(x,y) \in l_0$ and $x > x^*$. Denote the intersection point by $A(x^*, h_1)$. According to system (2.2), the trajectories hit the impulse set $M = \{(x,y)|y = h\}$ from below. Let $G = \{(x,y)|x > x^*,\ 0 < y \leqslant h_1\}$. The region G is divided into two parts G_1 and G_2 by the separatrix l_0, $G_1 = \{(x,y)|x > x^*,\ 0 < y \leqslant h_1,\ l_0 < 0\},\ G_2 = G - G_1$.

If $y^* < h < h_1$, then the trajectory passing through the point $B(x^*, h)$ comes from the region G_1. So there must exist p^* such that $B(x^*, h)$ jumps to the point B_1 and the curve segment $\overset{\frown}{BB_1}$ becomes the order-1 periodic solution denoted by l. When $p < p^*$, all the trajectories will tend to the equilibrium (x^*, y^*) after several impulsive effects at most. When $p > p^*$, the point B is mapped to the point B_2 by the impulsive effect. According to the proof of Theorem 3.1, there also exists a unique order-1 periodic solution.

If $h > h_1$, then the trajectory starting from the point A in the region G_2 will enter the region G_1 after several impulsive effects at most, and then all the trajectories will tend to the equilibrium (x^*, y^*) and system (2.2) has no order-1 periodic solution.

Proposition 3.3. *When $kb - bc - 2d \leq 0$ and $h > y^*$, if $y^* < h < h_1$, then there exists a harvest rate p^* such that system (2.2) has an order-1 periodic solution for $p \geqslant p*$ and no order-1 periodic solution for $p < p^*$. If $h > h_1$, then systems (2.2) has no order $k(k \geqslant 1)$ periodic solution.*

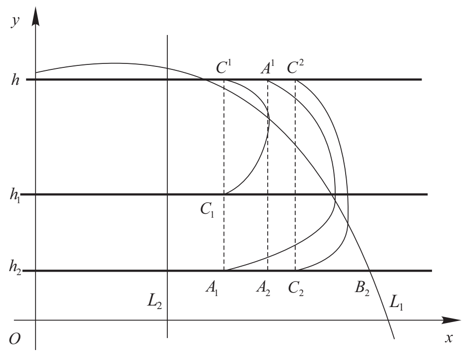

Fig. 4. The illustration of the effects of the harvest rate p on the position of order-1 periodic solution.

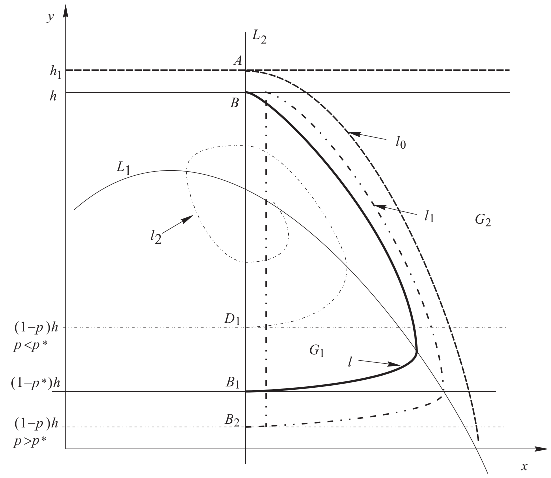

Fig. 5. The illustration of existence of order-1 periodic solution of system (2.1) for $y > y^*$.

4. Periodic solution for $kb - bc - 2d > 0$

When $kb - bc - 2d > 0$, we know from Lemma 2.4 that system (2.1) has a unique limit cycle. Denote the limit cycle by Γ_0. The limit cycle intersects the isoclinic line $L_2 : x = x^*$ at two point $A(x^*, h_2)$ and $B(x^*, h_3)$(see Fig. 6). The saddle sparatrix l_0 intersects the isoclinic line L_2 at the point (x^*, h_1). Clearly, $h_1 > h_2 > y^* > h_3$.

If $h < y^*$, then we know from the proof of Theorem 3.1 that system (2.2) has a unique order-1 periodic solution.

If $h > h_1$, then the trajectories of system (2.2) tend to the limit cycle after finite times impulsive effects at most for $t \to +\infty$ by Proposition 3.3.

If $h_2 < h < h_1$ and $(1-p)h > h_3$, then there is no order-1 periodic solution and the trajectories of system (2.2) tend to the limit cycle after finite times impulsive effects at most.

If $y^* < h < h_2$ and $(1-p)h < h_3$, system (2.2) has an order-1 periodic solution. When $h_3 < (1-p)h < h_2$, the trajectories of system (2.2) starting from the point (x, y), $0 < y < h_2$, $x > x^*$ will enter the interior of the limit cycle. So here only consider the trajectories lie in the interior of the limit cycle.

Theorem 4.1. *If $y^* < h < h_2$ and $h_3 < (1-p)h < h_2$, then system* (2.2) *has three kinds of order-1 periodic solutions.* (*see* Fig. 7).

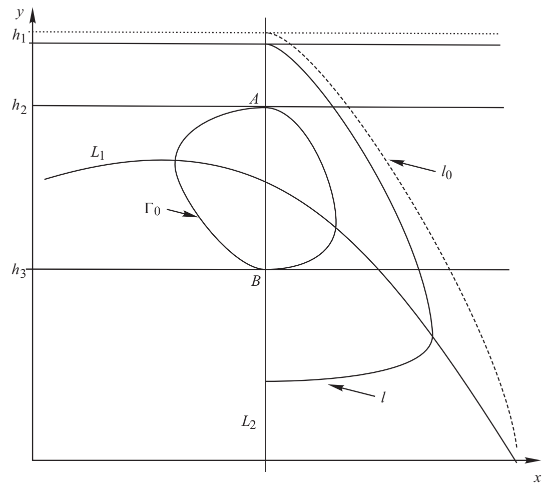

Fig. 6. The illustration of existence of order-1 periodic solution of system (2.1) for $kb - bc - 2d > 0$.

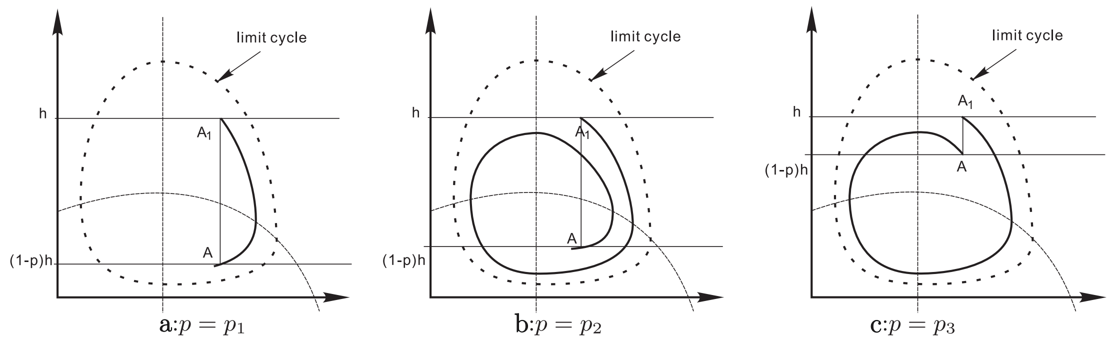

Fig. 7. Three kinds of order-1 periodic solution of system (2.2) for $kb - bc - 2d > 0,\ p_1 > p_2 > p_3$.

Proof. From the vector fields of system (2.1), we know that $dx/dt < 0$ when the trajectory passing through the isocline L_1 and $x > x^*$, so for $y^* < h < h_2$ and $h_3 < (1-p)h < h_2$, there must exist a point $A_1 \in M$ and p_1 $(h_3 < (1-p_1)h < h_2)$ such that system (2.2) has an order-1 periodic solution which consists of A_1, the image point $A \in N$ of A_1 and the trajectory between them (see Fig.7(a)).

Since the limit cycle is stable, then the trajectories starting from the point inside limit cycle tend to the limit cycle. For the point $A_1(x_{A_1}, h)$ close to the isocline L_2 sufficiently, the trajectory passing through the point $A_1(x_{A_1}, h)$ can intersect the line $x = x_{A_1}$ of the impulse function $\Delta x = 0$ at two points for $t \to -\infty$, then there exist p_2 and $p_3(p_2 > p_3)$ such that system (2.2) has an order-1 periodic solution, respectively, see Fig. 7(b) and (c). In particular, the trajectory between A and A_1 can revolve round the equilibrium (x^*, y^*) several cycles, which is similar to Fig. 7(b). This completes the proof. □

5. Numerical simulations

In order to verify the mathematical results given above, let $a = 1,\ b = 0.3,\ c = 2,\ d = 0.9,\ k = 6,$ then $bk - bc - 2d = -0.6 < 0$. It is easily known from Lemma 2.3 that system (2.1) has no limit cycle and the equilibrium $(x^*, y^*) = (3, 2.5)$ is a focus point, see the dot line in Fig. 8. Fig. 8(a) shows that there exists an order-1 periodic solution for $h = 2.4 < y^* = 2.5$ and $p = 0.6$. The trajectory starting from the initial point $(4, 1)$ tends to the order-1 periodic solution. Fig. 8(b) shows the change of position of the order-1 periodic solution with the harvest rate p varying. With the harvest rate increasing, the intersection point of order-1 periodic solution and the impulse set moves from left to right.

For the case of $bk - bc - 2d < 0$ and $h > y^*$, the numerical simulations can be seen in Fig. 9. Fig. 9(a) shows that there exists an order-1 periodic solution for $p = 0.6$. Fig. 9(b) show that there is no order-1 periodic solution for $p = 0.2$, the trajectory tends to the equilibrium (x^*, y^*) after several impulses. Therefore, there must exist p^*, $p^* \in (0.2, 0.6)$ such that Proposition 3.3 holds.

Let $a = 1,\ b = 0.3,\ c = 2,\ d = 0.4,\ k = 6$, then $bk - bc - 2d = 0.4 > 0$. It is easily known from Lemma 2.3 that system (2.1) has a unique limit cycle, see Fig. 10. Fig. 10(a) shows that there exists an order-1 periodic solution for

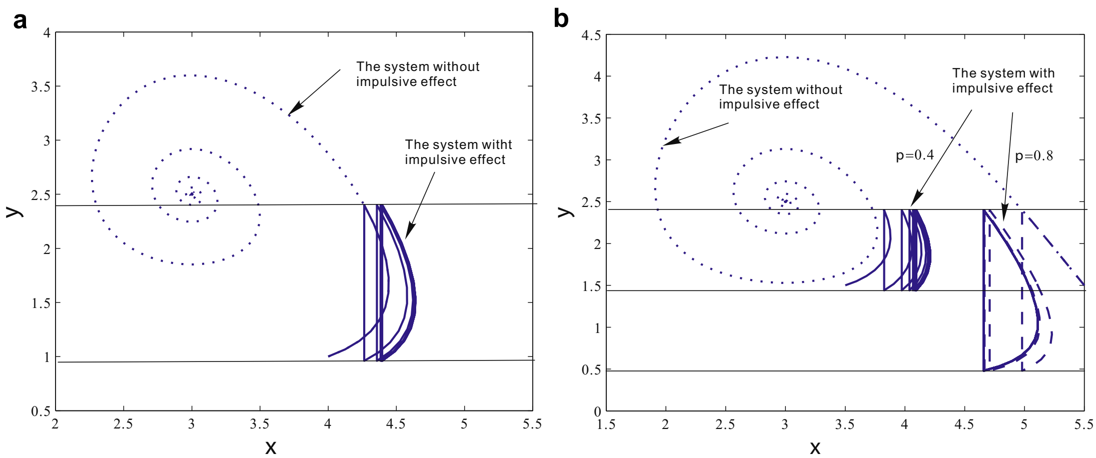

Fig. 8. The existence of order-1 periodic solution of system (2.1) for $h < y^*$.

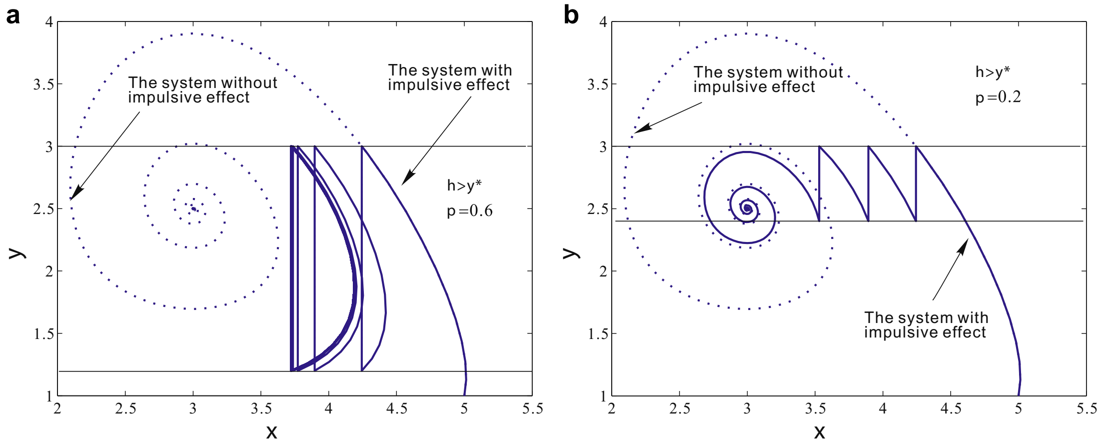

Fig. 9. The existence of order-1 periodic solution of system (2.1) for $h > y^*$.

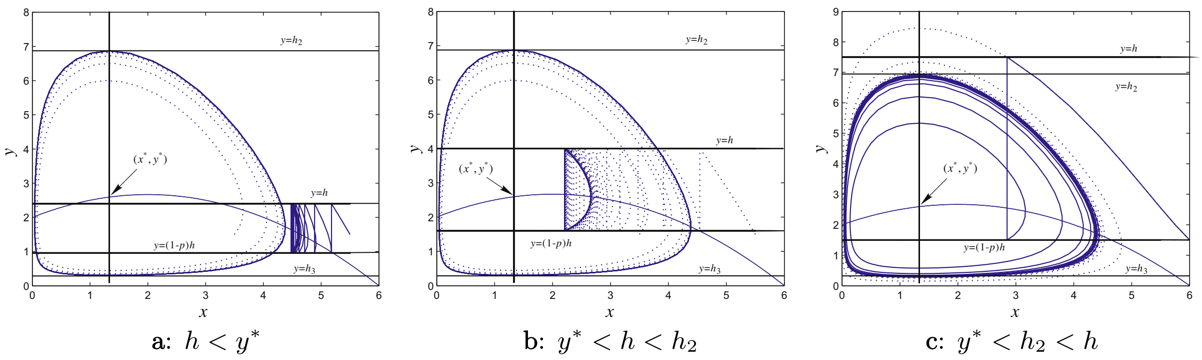

Fig. 10. The existence of periodic solution of system (2.1) for $bk - bc - 2d > 0$.

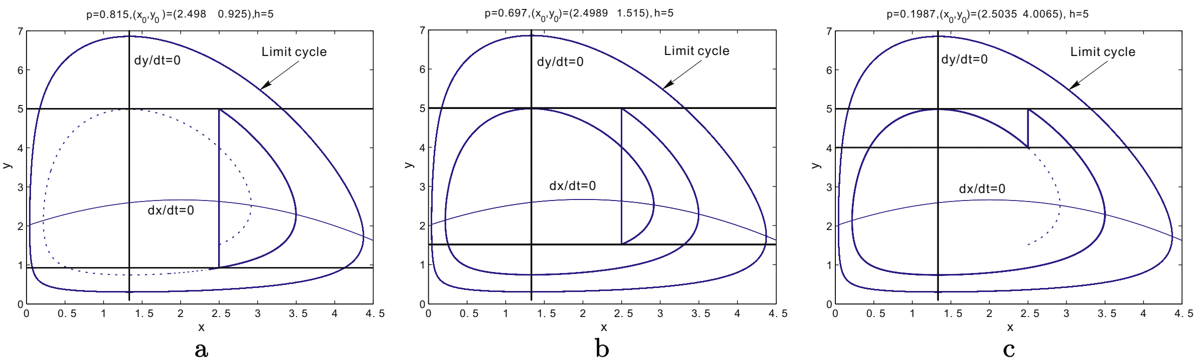

Fig. 11. The existence of three kinds of order-1 periodic solution of system (2.1) for $bk - bc - 2d > 0$, $y^* < h < h_2$ and $h_3 < (1-p)h < h_2$.

$h = 2.4 < y^* \doteq 2.59$ and $p = 0.6$. Fig. 10(b) shows that the system has an order-1 periodic solution for $h = 4 > y^*$ and $p = 0.6$. Let $h = 7.5$ and $p = 0.8$, then Fig. 10(c) shows that the trajectory tends to the limit cycle after one impulsive effect.

In Fig. 11, we can find that system (2.2) has three kinds of order-1 periodic solutions for $bk - bc - 2d > 0, y^* < h < h_2$ and $h_3 < (1-p)h < h_2$, where (x^*, h_2) and (x^*, h_3) are the intersection points of the limit cycle and the isocline $L_2 : x = x^*, h_2 > h_3$.

Theorem 4.1 only gives the existence of three kinds of order-1 periodic solutions for $y^* < h < h_2$ and $h_3 < (1-p)h < h_2$. If $p \neq p_1, p_2, p_3$, then the existences of order $k (k \geqslant 1)$ periodic solutions can be seen in Fig. 12. From Fig. 12, we can see that system (2.2) has order $k (k = 1, 2, \cdots, 6)$ periodic solutions for $y^* < h < h_2$ and $h_3 < (1-p)h < h_2$ respectively, which implies that system (2.2) has complex dynamical behaviors for $bk - bc - 2d > 0$ and $h > y^*$.

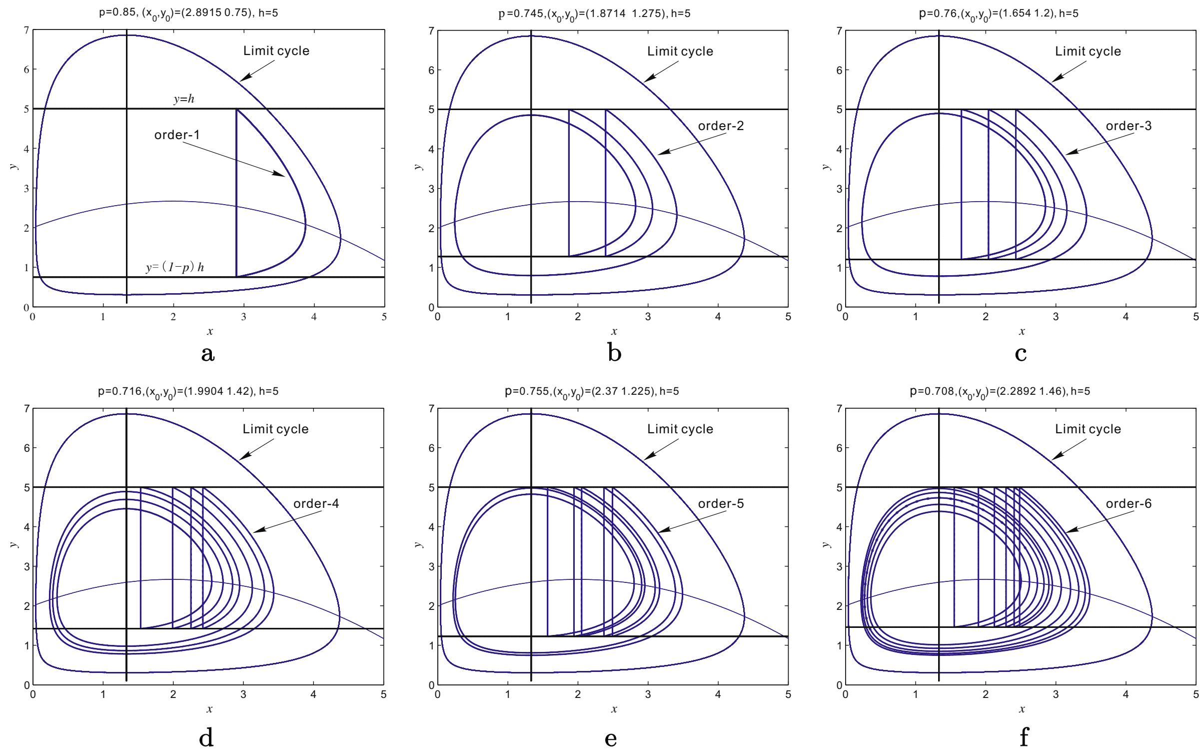

Fig. 12. The existence of order-$k(k = 1, 2, 3, 4, 5, 6)$ periodic solution of system (2.1) for $bk - bc - 2d > 0$, $y^* < h < h_2$ and $h_3 < (1 - p)h < h_2$.

6. Conclusions and discussions

In this paper,we have discussed the existence of periodic solution of the korean pine forest model with impulsive thinning measure. The results show that the model has order-1 periodic solution for $h < y^*$ which is orbitally asymptotically stable if Theorem 3.2 holds, and the existence of the periodic solution for $h > y^*$ or $bk - bc - 2d \leqslant 0$ needs some conditions to guarantee.

If $bk - bc - 2d > 0$, the existence of order $k(k \geqslant 1)$ periodic solution is complex. If $h < y^*$, there exists a unique periodic solution. If $h > y^*$, system (2.2) can have no periodic solution, three kinds of order-1 periodic solutions and order $k(k \geqslant 2)$ periodic solution. But there exist some troubles in the proof of the existences of order $k(k \geqslant 2)$ periodic solution, which be our next work.

Theses mathematical results show that, if the impulsive thinning measure is taken, the periodicity of the regeneration of the korean pine forest is related to the threshold level h of the seed trees and the harvest rate p. If the value of the threshold level is very big, system (2.2) has no order-$k(k \geqslant 1)$ periodic solution and the trajectories tend to the equilibrium or the limit cycle after finite times impulses at most, which implies that the impulsive thinning measure has no effects on the self-regeneration of the forest. If the threshold level and the harvest rate are given suitably, then system (2.2) has a unique order-1 periodic solution, the number of the young trees and the mature trees can be maintained periodically in certain level. Therefore, in practice, the suitable threshold level and the harvest rate should be given to maintain the sustainable and stable production of the forest.

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